STUDY OF THE PROPERTIES OF DENSE NUCLEAR MATTER AND APPLICATION TO SOME ASTROPHYSICAL SYSTEMS

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CERTIFICATE

This is to certify that the thesis entitled **" Study of the properties of dense nuclear matter and application to some astrophysical system"** which is being submitted by **PRADIP KUMAR SAHU** in partial fulfilment of the degree of Doctor of Philosophy in Science (Physics) of Utkal University, Bhubaneswar, is a record of his own research work carried out by him. He has carried out this investigations under our guidance. The matter embodied in the thesis has not been submitted for the award of any other degree by him or by anybody else.

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TO MY PARENTS

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List of Publications

1. Eigenfrequencies of radial pulsations of strange quark stars: by B. Datta, **P.K. Sahu**, J.D. Anand and A. Goyal: *Phys. Lett. B* **283** (1992) 313.

2. High density matter in the chiral sigma model: by **P.K. Sahu**, R. Basu and B. Datta: *Astrophys. J.* **416** (1993) 267.

3. Nuclear matter in the derivative scalar coupling model: Energy per nucleon and finite temperature equation of state: by B. Datta and **P.K. Sahu**: *Phys. Lett. B* **318** (1993) 277.

4. Masses of multiquark droplets: by L. Satpathy, **P.K. Sahu** and V.S. Uma Maheswari: *Phys. Rev. D* **49** (1994) 4642.

5. Neutrino emissivity of non-equilibrium quark stars matter: by **P.K. Sahu**: *Int. Jour. Mod. Phys. E* **2** (1993) 647.

6. On the limitations of neutrino emissivity formula of Iwamoto : by S.K.Ghosh, S.C. Phatak and **P.K. Sahu**: *Mod. Phys. Lett. A* (1994) (in press).

7. Quasi-molecular states in $^{12}C +^{16} O$ and $^{16}O +^{16} O$ systems: by L. Satpathy, **P.K. Sahu** and P.Sarangi: *J. Phys. G: Nucl. Part. Phys.* **18** (1992) 1793.

8. Quark stars in chiral colour dielectric model: by S.K. Ghosh and **P.K. Sahu.**: *Int. Jour. Mod. Phys. E* **2** (1993) 575.

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Contents

Chapter 1

Introduction

In ordinary stars, energy is lost through electromagnetic radiation from the surface. In order to accomplish this, a gradient in the temperature will be set up. The centre of the star will be relatively hot and the surface layers relatively cool. As energy is radiated away from the star, it shrinks just sufficient to provide the energy loss, and an equal amount gets added to the internally stored energy.

The contraction of the star leads to a continual heating of the stellar interior. A time will come when the temperature at the centre will rise to $\sim 10^7\ K$. At this stage, a new source of energy will appear in the interior : fusion of hydrogen nuclei into helium nuclei. Nuclear fusion releases enormous amounts of energy, which can prevent the star's gravitational collapse. The star attains hydrostatic equilibrium, and remains stable at a point on main sequence in the Herztsprung–Russel diagram, determined largely by its mass. The duration of this phase is very long (billions of years). That is why most stars that we see in the sky are main sequence stars.

In the above process, eventually a time will come when the supply of hydrogen in the star gets depleted. This leads to the following sequence of events : (1) a drop in the energy

production, (2) collapse of the star and a consequent rise of temperature in the core of the star (which is now mostly helium) and (3) fusion of helium to carbon and oxygen nuclei. This will lead to a second relatively stable phase of the star's life. However, unlike the first stage, a qualitatively new phenomenon now takes place. The energy release during the second phase of core collapse will not only lead to core heating but, in addition, cause an expansion of the outer layers of the star. As a result, the star gets a bloated shape (expected radius $\sim$ 100 million km), and the temperature of the outer layers drops. This is called a red giant. The helium burning phase is expected from theory to last several million years.

The depletion of helium again leads to a core contraction and a consequent rise in temperature. The further evolution of the star depends on the initial mass of the star.

According to stellar evolution theory, for stars whose initial mass is less than about 6 $M_{\odot}$ ($M_{\odot}$ = solar mass), the rise in temperature is insufficient to start the next fusion cycle (carbon $\longrightarrow$ neon). So, the collapse continues, and the energy released eventually blows off the remaining outer layers of the star. This produces a remnant stellar core and an expanding envelope. Such objects are called planetary nebulae, and some of these have been observed in the sky. The duration of this evolutionary phase is relatively short, about tens of thousands of years.

The remnant stellar core does not undergo gravitational contraction indefinitely. To understand this, one has to recall the Pauli exclusion principle in quantum mechanics. This principle states that no two fermions (*e.g.* electrons, protons, *etc.*) can possess identical quantum numbers (like spin, charge, angular momentum, *etc.*) at the same space-time point.

The remnant core contraction implies a high density situation for the material. That is, the constituents of the core are squeezed into smaller and smaller volumes of space. When the pressure brought on by the high density exceeds the electrostatic binding energy of the electrons to the atomic nuclei, the bound electrons will get detached from the nuclei and form a gas of free electrons. Because the electrons must obey the Pauli exclusion principle, the net result of the density squeeze is that the dense electron gas will behave as if it is a stiff ball of steel, *i.e.* it cannot be further compressed after a certain point. In other words, a dense electron gas will exert a pressure that will resist gravitational collapse. This pressure is not thermal but entirely quantum mechanical in origin, and is called degeneracy pressure. Configurations of the remnant core whose degeneracy pressure balances the gravitational attraction are called white dwarfs. At this configuration, the diameter is very small (to correspond to the requisite high density), about 1% of the solar diameter, *i.e.* comparable to the earth's diameter, and is very hot. From this point in time onwards, the white dwarfs gradually cool down. Because of the small surface area and the peculiarity of its composition, the time scale of cooling of a white dwarf is very large, several billions of years. Typical densities of white dwarfs are $(10^6 - 10^9)\ g\ cm^{-3}$.

For stars with initial mass in excess of $6M_\odot$, the core is sufficiently massive so that when it undergoes gravitational contraction at the end of helium burning stage, temperatures become so high that new fusion processes can occur. These produce heavier atomic nuclei : carbon, oxygen, neon, *etc.* ... , upto iron. By the time iron nuclei are produced, there is a huge build-up of Coulomb repulsive forces because of the presence of a large number of positively charged protons. Furthermore, the binding energy per particle is very high for iron nuclei, so that these nuclei are very stable. As a result, no further fusion reactions

are possible, and the core starts to collapse again.

This time around, the gravitational collapse becomes too strong (because of higher mass), and will overcome the electron degeneracy pressure. Two final stages of this collapse are possible : neutron stars and black holes.

According to detailed calculations, when the density of matter in the remnant core reaches about 10^{12} $g\ cm^{-3}$, the composition is substantial amount of free neutrons (that is, unbound to nuclei), in addition to nuclei and the dense ambient electron gas. These neutrons will not undergo beta decay because of phase space barrier brought on by the presence of dense surrounding electrons, whose Fermi momentum gets to be pretty high ($\geq$ 20 MeV). Beyond a density of about 10^{14} $g\ cm^{-3}$, the nuclei 'dissolve' because of depletion of their proton concentration due to inverse beta decay (which becomes energetically favourable at high densities), and the composition of the core becomes mostly neutrons, with a small admixture of protons and electrons. If the core is not too massive (calculations suggest an upper limit of 2-3 solar masses), the degeneracy pressure and the repulsive force of neutron matter can balance the gravitational attraction. A stable configuration is then possible, and it is called a neutron star. Because of the high densities involved, a neutron star is expected to be very compact by stellar standards, with radius about (10-15) km. Since degeneracy pressure plays an important role for the stability of white dwarfs and neutron stars, these are sometimes referred to as degenerate stars (in which all fusion reactions have stopped, unlike ordinary stars).

If the mass of the collapsing core is more than $3M_{\odot}$, the collapse will go unchecked. There is no physical mechanism known that can provide enough repulsive forces to halt the collapse. This will lead to a singularity situation, where the density will be extremely

high with a large surface gravity such that not even light radiation can emerge from its surface. This is called a black hole.

Immediately after the discovery of neutron in 1932, it was visualized by Landau that very compact stable objects, made up entirely of neutrons could be formed in stellar collapse. Baade and Zwicky [1] in 1934 suggested that such dense objects could be the remnants formed in the aftermath of supernova explosions. The first quantitative theoretical estimates for the bulk properties of neutron stars, such as mass and radius, were given by Oppenheimer and Volkoff [2]. Although the existence of neutron stars were predicted in 1932 and the theoretical estimates of their mass and radius were done in 1939 (Oppenheimer and Volkoff [2]), it was not until the discovery of pulsars [3] in 1968, that the real astrophysical importance of the neutron star idea was established. It is now generally accepted that pulsars are rotating, magnetized neutron stars. In order to have a proper understanding of the structure and the dynamic of pulsars, it is necessary to study the structure and composition of neutron stars in detail. Besides, the maximum mass possible for stable neutron stars is important in another astrophysical situation, namely the identification of a possible black hole in compact binary systems.

Typical neutron star mass and radius are expected to be about $1 M_\odot$ ($M_\odot$ = solar mass = 2×10^{33} gm) and 10 km respectively. This means that the average density inside a neutron star is of the order ρ_0 or higher, where ρ_0 = nuclear matter density $\simeq 2.8 \times 10^{14}$ g cm^{-3}. Most of the matter inside a neutron star is expected to be at densities much above ρ_0. The behaviour of such matter is not well understood. From heavy-ion collision data at intermediate energies, one now hopes to derive reliable nuclear equation of state for nuclear matter for $\rho = (2-3)\rho_0$ [4]. The interactions that will be

important beyond this density are not known, although several theoretical models have been proposed during the last twenty years. An additional difficulty is the many-body aspect of the problem. A proper way to incorporate the many-body correlation effect in high density strongly interacting matter is necessary, but there exists as yet no consensus on this subject.

There is general acceptance that neutron star interiors can be divided into four distinct density regimes characterized by different compositions [5, 6] :

1. The density at the surface made up basically of ^{56}Fe nuclei bound in a lattice immersed in a sea of relativistic nondegenerate electrons, which ranges from 7.86 $g\ cm^{-3}$ to a density of $\sim 10^7\ g\ cm^{-3}$.

2. The second density region is between $10^7\ g\ cm^{-3}$ to $4.3 \times 10^{11}\ g\ cm^{-3}$. The latter corresponds to the neutron drip point, at which point onwards continuum neutron states start getting populated.

3. The third density region starts at $4.3 \times 10^{11}\ g\ cm^{-3}$ and it continues all the way upto nuclear matter density of $\rho_0 = 2.8 \times 10^{14}\ g\ cm^{-3}$. In this density regime, the matter consist of neutron–rich nuclei, neutrons (expected to be superfluid) and small number of protons and electrons.

   The density regions (1), (2) and (3) together define the crust of the neutron star because the nuclei are expected to be in lattice structure. The crustal depth from the surface depends on the equation of state and roughly corresponds to about $(10 - 15)\%$ of radius.

4. The fourth density regime corresponds to $\rho \geq \rho_0$. The central densities can be $\simeq 10\ \rho_0$.

Oppenheimer and Volkoff [2] considered neutron star matter to be non-interacting neutron matter and obtained a value $0.7M_\odot$ as the maximum gravitational mass and a corresponding radius $\sim 17\ km$. Today it is believed that neutron star interiors are made up asymmetric nuclear matter, with perhaps small admixture of pions and hyperons. Since neutron star matter contains matter at density higher than the nuclear matter density, one should take into account, as far as possible, the nuclear forces at short ranges. In the last twenty years, many attempts have been made to calculate the equation of state $p = p(\rho)$, p = pressure of high density matter, taking into account the nuclear forces at short distances. Most such realistic calculations give the neutron star maximum gravitational mass to be between $(1.8 - 2)M_\odot$.

Clearly, it is important that interactions among neutrons be included in any realistic calculation of neutron star structure. The equation of state where the density $\geq 10\ \rho_0$ is poorly understood due to uncertainty in nuclear interactions, and has been a focus of much theoretical research in recent years. In this thesis, we make a study of properties of dense nuclear matter using a field theoretical approach, which has gained increasing importance in the last few years.

This thesis contains two parts. Part one (chapters 2–5) deals with the nuclear equation of state based on relativistic mean field theory. In chapter 2, we briefly review the equations of state in both non-relativistic and relativistic approaches. In chapter 3, we make a detailed study of the chiral sigma model and use it to derive an equation of state of asymmetric nuclear matter. Recently, Datta and Alpar [7] showed that the Vela pulsar

glitch data suggest stiffer rather than softer equation of state for neutron star matter. The chiral sigma model seems to possess such a desirable property at high densities. We have extended the chiral sigma model calculation to finite temperatures in chapter 4. We also discuss the derivative scalar coupling model proposed by Zimanyi and Moszkowski [8] some years ago. In chapter 5, we examine in the context of our model (as discussed in chapter 3), the question of a possible phase transition from hadronic matter to quark matter at high densities. We discuss the formation of strangelets of large mass in quark matter at high densities based on non-relativistic treatment in chapter 6. The detection of strangelets may be the most unambiguous way to confirm the formation of quark-gluon plasma in heavy ion collision experiment. The study of this chapter gives basic idea on the strangeness contents in the high density matter such as core of the neutron stars.

In the second part of the thesis, we consider certain astrophysical applications. These are the modelling of stable neutron star structure, radial oscillations of quark stars and cooling rates of neutron stars if these objects possess ($u,\ d,\ s$) quark matter in their cores. The structure of neutron stars, quark stars and quark star oscillations are discussed in chapter 7. In the chapter 8, we discuss the limitation of previously suggested neutrino emissivity formula for the quark matter in the neutron stars. In the last chapter, we summarize the highlights and the main points of this thesis.

Chapter 2

The equation of state of matter in neutron star interior : a review of previous work

2.1 Introduction

The equation of state $p(\rho)$ (p=pressure, ρ=total mass-energy density) above the nuclear matter density ($\rho > 3 \times 10^{14}\ g\ cm^{-3}$) plays a crucial role to determine the equilibrium structure of neutron stars. A considerable amount of work has already been done in the last two decades on this subject. In order to set a proper perspective for the discussion in the following chapters, we give here a brief review of previous work on equations of state of matter in neutron star interior. Upto about nuclear density the equation of state is reasonably well known, but the central density of neutron star can be almost an order of magnitude higher. In this regime, the physics is unclear. Below nuclear density the nuclear gas is dilute and one can use the perturbative methods, while at

much higher densities we would be in the asymptotically free region of quantum-chromo-dynamics. Neutron star matter is usually divided into four general density regions [5, 6]:

1. Near the surface region, where density is upto $10^7\ g\ cm^{-3}$, a lattice of bare nuclei (mainly ^{56}Fe) is immersed in a gas of relativistic and degenerate electron gas. The equation of state in this region is influenced by temperature and magnetic fields . But it hardly matters to the structure of neutron stars.

2. The next region is the neutron-rich nuclei upto ^{118}Kr, where the protons inside the nuclei undergo inverse beta decay. Due to the inverse beta decay, the electron gas occupies the lowest Fermi energy level, which allows the system to be in a lower value for the ground state energy.

3. The third density region begins at about $4.3 \times 10^{11}\ g\ cm^{-3}$ is called the neutron drip point. In this region, some of the neutrons in the nuclei get detached from the parent nucleus. These neutrons are unbound and stable.

4. In the final region, (density $> 2.8 \times 10^{14}\ g\ cm^{-3}$), the individual nuclei merge into each other. So, the composition of the matter is expected to be a fluid of almost uniform neutron matter together with protons, electrons and possibly muons, pions, hyperons *etc*. Above this density, there is possibility of a phase transition from neutron matter to $(u,\ d,\ s)$ quark matter.

Because of large neutrino emissivity, neutron star matter is cold and degenerate. Therefore, we do not have to worry about temperature in deriving equation of state.

The first three of the above density regions are reasonably well-understood. Comprehensive account of the physics of these regions are given by Canuto [5] and Baym and

Pethick [6]. All versions of equation of state of the fourth region (which is above the nuclear matter densities) differ from each other, due to the inadequate knowledge of nuclear interactions as well as the lack of proper many–body techniques, that will be relevant to describe neutron star matter. In what follows, we briefly review the two major techniques that have been used :

1. non-relativistic theories,

2. relativistic field theoretical model.

2.2 Non-relativistic approach

In this approach one uses two-body potentials which are fitted to the nucleon-nucleon scattering data, as well as a three-body term. The form of the potential is chosen whose parameters are determined using the data of few-body nuclei and saturation properties of nuclear matter. For a non-relativistic model, the starting point is the two-body potential. For the many–body method, there are two approaches, Brueckner-Bethe-Goldstone theory [9] and the variational method [10]. Since the major constituents of matter inside the neutron stars are neutrons, we concentrate our attention on neutrons and their interactions. The experimental information regarding nucleon-nucleon scattering data and known properties of deuteron do not uniquely determine the nucleon-nucleon potential. Hence, for setting up the equation of state, it is required to have fit with known properties of equilibrium nuclear matter at saturation density such as binding energy per nucleon, compression modulus *etc.* Before we discuss the recent non-relativistic equation of state given by Wiringa et al. [11] that provides a reasonably good description, we briefly review

the historical equations of state at high densities in the following sections.

I. Reid Model:

The Reid model [12] is based on phenomenological nucleon-nucleon potentials, which fits the scattering data very well. It has been used extensively in the calculation of neutron star structure (reviews by Baym and Pethick [13]; Canuto [14]). The calculation of equation of state $p(\rho)$ using the lowest order constrained variational method seems to be accurate enough for the central part of the Reid potential (Pandharipande and Bethe [15]). But Pandharipande and Wiringa [16] calculated the nuclear matter properties using Reid potential and found that both the equilibrium density as well as binding energy were too large. Hence, this model is now considered to be unrealistic.

II. Bethe-Johnson model:

The short-range interaction between nucleons is not uniquely determined by the nucleon-nucleon scattering data. So, Bethe and Johnson [17] proposed a phenomenological potential model where they suggested several different potential models for nucleon-nucleon interaction, assuming various plausible strengths for short-range repulsion by fitting the scattering data. The maximum mass calculations for neutron stars with this equation of state are given in Malone *et al*. [18]. At high densities the constituents of matter are made up of nucleons (N) and hyperons (Y). However, the hyperonic interactions are not included in full details.

These two models (I and II) are phenomenological density independent static potential models. According to these, the nuclear matter energy at high densities increases linearly with the density.

III. Tensor Interaction model:

The attraction between nucleons which comes from higher order contribution of the pion exchange tensor interaction was studied by Pandharipande and Smith [19]. Their work is a generalization of various tensor interaction models proposed earlier by Green and Haapakoski [20]. These interactions fit only the s-wave scattering data, and differ mainly in the strength of the short-range repulsion for which a specific form is presumed. The work by Smith and Pandharipande in [21] suggests that the low energy nucleon-nucleon scattering data can be explained by attributing all the attraction between nucleons to tensor interaction. It has been seen that the calculation with tensor interaction using lowest order variational and Brueckner methods (Green and Niskanen [22]) satisfies only half of the nuclear matter binding energy at saturation density. Thus the tensor interaction model can not explain the nuclear matter properties in a satisfactory way.

IV. The mean field model:

Pandharipande and Smith [23] assumed a model, called the mean field model, which states that the attraction between nucleons is due to the exchange of an effective scalar meson. The quadrupole moment of the deuteron as well as the phase shift in the ${}^3P_0$, ${}^3P_1$ and ${}^3P_2$ channels clearly indicate the presence of the one-pion exchange tensor force between the nucleons. However, a detailed analysis of the attractive interaction due to all possible tensor potentials (Smith and Pandharipande [21]) suggest that it is almost independent of the spin and isospin of the interacting nucleons, and thus its contribution in matter could be similar to that due to coupling of nucleons to a scalar field. The scalar field, treated in the mean field approximation, was used by Walecka [24], which we shall discuss later in detail. Following the Walecka model, the nucleons moving in a

mean scalar field are assumed to interact by a central potential generated by ω, ρ and π mesons exchanges. The potential approximation may be appropriate for ω and ρ vector fields, while the central parts of the pion exchange potentials have a negligibly small effect. The mean field approximation is unjustified for the vector field because the ω-ρ exchange potential have a range $\sim 0.2\ fm$, which is much smaller than the mean interparticle spacing $\sim 1.2\ fm$. The short-range corrections induced by the ω-ρ exchange potentials are treated by variational method using a hypernetted chain formalism (Pandharipande and Bethe [25]). The coupling constant of ω, ρ and σ are calculated from the nuclear matter binding, symmetry energy and the equilibrium density. The incompressibility parameter obtained by this theory is $\sim 310\ MeV$. The nucleon-nucleon scattering data can not be explained by an interaction which has been used in the mean field model. But this model satisfies all the empirically computed properties of nuclear matter.

The attraction between nucleons in the above two models, *i.e.*, tensor and mean field decrease with increasing density. However, this is the general characteristic of microscopic models, which are based on the mean field theoretic calculations. The draw back of these two models is that at small densities the energy is proportional to the density, whereas at higher densities it tends to saturate.

V. Friedman and Pandharipande model:

Friedman and Pandharipande [26] gave a model for the equation of state of dense neutron and nuclear matter, where they used the variational method as suggested earlier by Pandharipande [10] to calculate the equation of state for a wide range of density. In their model, they used improved phenomenological nucleon-nucleon interactions [27, 28]. The two nucleon interaction has short–range and intermediate range parts, and also

possesses the pion–exchange contribution. There is also a contribution due to a three–nucleon interaction which has a complicated form (it is a function of strength parameters, the interparticle distance and also the alignment angles). The parameters involved in the three–nucleon interactions are determined by reproducing the equilibrium density, energy and incompressibility of nuclear matter based on variational calculations. This model fits well the nucleon-nucleon scattering cross-section data, the deuteron properties and the nuclear matter properties.

VI. Wiringa, Fiks and Fabrocini model:

Wiringa *et al.* [11] proposed a model which is most firmly based on available nuclear data. This model improves on the earlier work by Friedman and Pandharipande [26]. In this model the two-nucleon potential is taken to be the Argonne v14 (AV14) or Urbana (UV14) potential. Both have the identical structure, but differ in the strength of the short-range tensor force. They are called v14 models because they are the sum of v14 operator components (like $\sigma_i.\sigma_j,\ \pi_i.\pi_j$, *etc.*). Each of these components has three radial pieces which includes the long-range one-pion exchange, an intermediate-range part that comes from the two-pion exchange processes, and a short-range part, coming either from the exchange of heavier mesons or overlaps of composite quark systems. All the free parameters are fitted to nucleon-nucleon scattering data and deuteron properties.

For the three-nucleon interaction, the Urbana VII potential is used, which has a two-pion exchange part and intermediate-range repulsive contribution. Calculations have been carried out for the Urbana v_{14} plus three-nucleon interaction (TNI) model of Lagaris and Pandharipande [29].

The many-body calculations are based on the variational principle where one uses

the technique called the Fermi hypernetted chain-single-operator chain (FHNC − SOC) integral equations [27, 30].

The authors obtained the nuclear matter saturation properties such as binding energy, saturation density and incompressibility parameter value of nuclear matter at saturation density are –16.6 MeV, 0.157 fm^{-3} and 261.0 MeV respectively for UV14 plus TNI model. This is a reasonable improvement over all the other non-relativistic approaches.

In their study, one notices that the sound velocity s in the medium given by the equation of state (in parametric form) for beta-stable ($n,\ p,\ e$ *i.e.*, no hyperons) matter based on non-relativistic approach violates causality above $\rho = 1 fm^{-3}$. This is an undesirable feature of this method at high densities. However, they predicted the maximum neutron star mass to be $2.2 M_\odot$ and for $1.4 M_\odot$ neutron star, the central density turns out to be substantially below $1 fm^{-3}$ which is quite realistic. It may be noted that the modern potentials supplemented with reasonable three-body interactions yield very similar models of neutron star structure parameters. For the $1.4 M_\odot$ models, the radius is $\simeq 10.4 - 11.2\ km$, and the central density is about $6\rho_o$.

2.3 The relativistic approach

The shape of baryonic potential is not known at very small interparticle separations ($\leq 0.5 fm$). Also, it is not clear that the potential description will continue to remain valid at such short ranges. Moreover, in the neutron star interiors, the Fermi momentum of the degenerate neutron is large. Therefore, the non-relativistic approach may not be adequate. In the recent times, the relativistic approach has drawn considerable attention.

In the relativistic approach, one usually starts from a local, renormalizable field theory with baryon and explicit meson degrees of freedom. These models have the advantage of being relativistic, however, one drawback is that one does not know how to relate it to nucleon-nucleon scattering data. The theory is chosen to be renormalizable in order to fix the coupling constants and the mass parameters by empirical properties of nuclear matter at saturation (binding, density, compression modulus, effective mass and symmetry energy). As a starting point, one chooses the *mean field approximation* (MFA) which should be reasonably good at very high densities (a few times nuclear) [31]. In the second step, one includes one-loop vacuum fluctuations which leads to what is called the *relativistic Hartree approximation* (RHA) [32]. This approach is currently used as a reasonable way of parametrizing the equation of state.

The main features of this approach can already be seen in a simple model. Assuming that a neutral scalar meson field (ϕ) and a neutral vector field (V_μ) couple to the baryon current by interaction terms of the form

$$g_s\bar{\psi}\psi\phi \;\; and \;\; g_v\bar{\psi}\gamma^\mu\psi V_\mu, \tag{2.1}$$

the Lagrangian can be written as ($\hbar = 1 = c$)

$$\begin{aligned}\mathcal{L} = \bar{\psi}[\gamma^\mu(i\partial_\mu - g_v V_\mu) - (M - g_s\phi)]\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) \\ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu;\end{aligned} \tag{2.2}$$

where

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu. \tag{2.3}$$

From Euler-Lagrange equation

$$\frac{\partial}{\partial x^\mu}[\frac{\partial \mathcal{L}}{\partial(\partial q_i/\partial x^\mu)}] - \frac{\partial \mathcal{L}}{\partial q_i} = 0 \tag{2.4}$$

where q_i is one of the generalized coordinates, one yields the field equations

$$(\partial_\mu \partial^\mu + {m_s}^2)\phi = g_s \bar{\psi}\psi,$$

$$\partial_\mu F^{\mu\nu} + {m_v}^2 V^\nu = g_v \bar{\psi}\gamma^\nu \psi,$$

$$[\gamma^\mu(i\partial_\mu - g_v V_\mu) - (M - g_s\phi)]\psi = 0. \tag{2.5}$$

The MFA consists in replacing the meson-field operators by their expectation values :

$$< \phi >\equiv \phi_o, \quad < V_\mu >\equiv \delta_{\mu o} V_o, \tag{2.6}$$

if we consider a static uniform system. Then, the meson field equations become

$$\phi_o = \frac{g_s}{m_s^2} < \bar{\psi}\psi >\equiv \frac{g_s}{m_s^2}\rho_s, \tag{2.7}$$

$$V_o = \frac{g_v}{m_v^2} < \psi^\dagger\psi >\equiv \frac{g_v}{m_v^2}\rho_B. \tag{2.8}$$

In this approximation, the nucleon field operator satisfies a linear equation :

$$[-i\gamma^\mu \partial_\mu + g_v \gamma^o V_o + M^*]\psi = 0, \tag{2.9}$$

where $M^* = M - g_s\phi_o$ is the effective mass of the nucleon and the Lagrangian density takes the form

$$\mathcal{L}_{\mathcal{MFT}} = \bar{\psi}[i\gamma_\mu \partial^\mu - g_v \gamma^o V_o - M^*]\psi - \frac{1}{2}m_s^2\phi_o^2 + \frac{1}{2}m_v^2 V_o^2. \tag{2.10}$$

The quantization here in this case is straightforward. In the mean field approximation, the conserved energy–momentum tensor is

$$T_{\mu\nu} = -g_{\mu\nu}\mathcal{L}_{\mathcal{MFT}} + \frac{\partial\psi}{\partial x^\nu}\frac{\partial\mathcal{L}_{\mathcal{MFT}}}{\partial(\partial\psi/\partial x_\mu)}. \tag{2.11}$$

Assuming that the bulk neutron star matter can be treated as a uniform and perfect fluid, we can write the energy density of the system as

$$\epsilon =< T_{oo} > \tag{2.12}$$

and the pressure as

$$p = \frac{1}{3} < T_{ii} > . \tag{2.13}$$

The ground state of nuclear matter is characterised by Fermi momentum k_F which is related to the baryon density ρ_B as :

$$\rho_B = \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k = \frac{\gamma}{6\pi^2}k_F^3. \tag{2.14}$$

Here $\gamma = 4$ for symmetric nucleon matter, $\gamma = 2$ for neutron matter. The mean field ϕ_o(or M^*) is determined self-consistently as

$$M^* = M - g_s\phi_o = M - \frac{g_s^2}{m_s^2}\rho_s = M - \frac{g_s^2}{m_s^2}\frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k\frac{M^*}{E^*(k)}, \tag{2.15}$$

where $E^*(k) = (k^2 + M^{*2})^{1/2}$. This leads to a transcendental equation for a given $k_F(\rho_B)$. It turns out that only the ratio g_s^2/m_s^2 and g_v^2/m_v^2 enter in the equation of state. These parameters are fixed such that the nuclear matter will give rise to the correct binding energy and saturation density :

$$(\frac{E - BM}{B})_o = -15.75 \text{ MeV}, \tag{2.16}$$

$$k_F^o = 1.42\ fm^{-1}\ (\gamma = 4). \tag{2.17}$$

Thus

$$C_s^2 \equiv g_s^2(\frac{M^2}{m_s^2}) = 267.1, \tag{2.18}$$

$$C_v^2 \equiv g_v^2(\frac{M^2}{m_v^2}) = 195.9. \tag{2.19}$$

With these values one can compute the equation of state for neutron matter. In this simple model there is a first order phase transition (similar to the liquid-gas transition in the van der Waals' equation of state). At asymptotically high densities, the velocity of sound approaches the velocity of light.

One can make the model more realistic by including for instance, the interaction of ρ-mesons (charged vector mesons). In such model (QHD-II of Serot and Walecka [32]) the ρ meson stiffens the equation of state at relatively low density and causes the gas-liquid phase transition to disappear.

The above field theoretical treatment is now-a-days referred to as the Walecka model [24]. In 1974, Walecka first proposed this mean field model, where he chose the coupling constants in such a way that it fitted the nuclear matter binding energy and saturation density. In this model, the value of nuclear matter incompressibility at saturation is quite high. The isospin triplet vector meson ρ, was not included, but is of relevance in neutron star interior $(n,\ p,\ e)$ matter. The extension of this model will be discussed in the next section.

I. Glendenning Model:

Glendenning [31] presented a relativistic field theoretical model for densities near as well as above the nuclear matter density. This includes isospin-asymmetric baryon matter.

In this model was included the nucleons, the mesons : ω, ρ and π, e^- and μ^- particles along with the self-interaction of the scalar meson field σ. The self-interaction of σ-field was chosen to be of the form :

$$U(\sigma) = (\frac{b}{3}m_n + \frac{c}{4}g_\sigma\sigma)(g_\sigma\sigma)^3 \tag{2.20}$$

where m_n is the nucleon rest mass and b, c are parameters. The coupling constants and the parameters b, c were determined by satisfying the following bulk properties of nuclear matter *i.e.*,

saturation particle density = 0.145 fm^{-3}

saturation binding energy = -15.96 $MeV/nucleon$

asymmetry energy = 37 MeV

and the incompressibility = 280 MeV

The author chose two different sets of parameters, one of which gives a soft and other a stiff equation of state at high densities, for fits to nuclear physics.

In 1985, Glendenning [33] extended the above formalism to derive an equation of state of hyperonic matter. He used the same constraints as earlier to fix the various parameters. Here, he included the K and K^* meson exchanges and also the self-interaction form as earlier. The coupling strengths of the isobar Δ and the hyperons (Λ, $\Sigma^{\pm,0}$, $\Xi^{\pm,0}$) to the mesons are taken from the work of Moszkowski [34]. This work indicates that cores of the heavier neutron stars are dominated by hyperons and the total hyperon population for such stars are $15\% - 20\%$, depending on whether pions condense or not. The advantage of this model is that one can make the equation of state soft or stiff depending on the input parameters, namely the incompressibility value. In this case the following four separate

hyperonic equation of state matter are considered.

Case 1 : n, p, hyperons, Δ, e^-, μ^-, π^-;

Case 2 : n, p, hyperons, Δ, e^-, μ^-;

Case 3 : like case 2 but with universal coupling of hyperons;

Case 4 : like case 3 but no ρ- exchange ($g_\sigma = 0$).

We describe now the results of a more recent analysis [35] for the following ($\sigma,\ \omega,\ \rho$) model, whose Lagrangian is given as

$$\begin{aligned}\mathcal{L} = \sum_B \bar{\psi}_B(i\gamma^\mu\partial_\mu - m_B - g_{\sigma B}\sigma - g_{\omega B}\gamma^\mu\omega_\mu - \frac{1}{2}g_{\rho B}\gamma^\mu\tau_3\rho_\mu^{(3)})\psi_B \\ +\frac{1}{2}(\gamma_\mu\sigma\gamma^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ -\frac{1}{4}\vec{\rho}_{\mu\nu}\vec{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu.\vec{\rho}^\mu - \frac{1}{3}bm_n(g_\sigma\sigma)^3 - \frac{1}{4}c(g_\sigma\sigma)^4. \qquad (2.21)\end{aligned}$$

Here the sum runs over the baryons $N,\ \Lambda,\ \Sigma,\ \Xi,\ \Delta,\ etc.$

For uniform matter, the theory depends only on the ratios $g_\sigma/m_\sigma,\ g_\omega/m_\omega,\ g_\rho/m_\rho$ and the self-interaction coefficients $b,\ c$. These five quantities are determined empirically, in both the MFA and the RHA (Table 1 in Ref. [35]). From this calculation one notices that the equation of state for stable neutron star matter will be softer than the pure neutron matter. This softening is the result of the replacement of energetic neutrons by hyperons at rest. It is most remarkable that the maximum allowed neutron star mass depends very sensitively on this coupling. This point has been further investigated in Ref. [36], with the conclusion that even if we have a perfect knowledge of the nuclear equation of state upto about $2\rho_o$, there is still a large uncertainty in the maximum neutron star mass because of our lack of knowledge of the hyperon interactions. This is one of the most relevant "interface" of astrophysics and nuclear physics. However, further laboratory experiments

are needed to reduce the large uncertainties.

II. Alonso and Cabanell model:

Alonso and Cabanell [37] gave a model in which the nucleons interact via scalar (σ) mesons, pions (π), and vector (ω and ρ) mesons. The model is solved in the renormalized Hartree approximation. The authors give two sets of equation of state (I, II).

The equation of state I is derived by fitting the properties of symmetric nuclear matter at nuclear density in a satisfactory way. This is in good agreement with the equation of state obtained by Baym, Bethe and Pethick [38] in the region above the neutron drip for neutron matter. The value of the nuclear incompressibility obtained in this model is too high (460 MeV). This large incompressibility is perhaps due to the absence of σ-self-interaction term in the Lagrangian.

The equation of state II is derived using the chirally invariant σ- model Lagrangian, coupled to ω and ρ mesons and having an explicit symmetry breaking term. This equation of state fits accurately all the known properties of symmetry energy of nuclear matter at nuclear density. The value of the nuclear incompressibility at saturation density comes out to be 225 MeV.

For these two equations of state, the total number of adjustable parameters are rather large (12 for set I and 15 for set II). A peculiar feature of these models is that both (I, II) show a dip in equation of state. These dips do not represent any phase transition for the system.

III. Chiral sigma model (Glendenning):

Chiral symmetry is a good hadron symmetry, ranking only below the isotopic spin symmetry [39]. For this reason, it is desirable to have chiral symmetry in any theory of

dense hadronic matter. At the same time, the theory should be capable of describing the bulk properties of nuclear matter. But a theory that satisfies both the conditions is not available.

In 1986, Glendenning [40] derived the equation of state based on the mean field approximation. Here, the equation of motion for the mean fields is also derived in the general case for the chiral sigma model supplemented by a gauge massless vector meson in interaction with the other hadrons, including baryon resonances. Here, omega meson is considered with dynamical masses. In this theory, the gauge field ω_μ of massless vector meson is introduced into the chiral sigma model through the covariant derivatives. Moreover, the linear term of σ field in it is the symmetry breaking term, by which the pion acquires a finite mass. In addition to the ω-mesons, the theory has scalar meson and the pseudoscalar pion. This theory has two parameters, which are determined from the saturation density and binding energy per nucleon in normal symmetric matter. The value of the incompressibility is rather very large *i.e.*, $K = 650\ MeV$.

In a subsequent paper, Glendenning [41] extended the chiral sigma model based on the mean field approximation where the ω-meson does not have a dynamically generated mass, even if one considers the vacuum renormalization correction to the theory. In this model, he considered only the normal non-pion condensed state of matter and included hyperons in beta equilibrium with nucleons and leptons. He fitted the parameters of the theory to obtain two equation of state corresponding to two compression moduli, a "stiff" ($K = 300\ MeV$) and a "soft" ($K = 200\ MeV$), by reproducing correct nuclear matter properties.

IV. Chiral sigma model (Prakash and Ainsworth):

Prakash and Ainsworth [42] proposed an equation of state based on the chiral sigma model. They examined the role of the many-body effects provided by the chiral sigma model in the equation of state of symmetric nuclear matter and neutron rich matter. They include the σ- meson one-loop contributions, but the isoscalar vector field is not generated dynamically, so that its role is reduced to an empirical one. A set of equation of state is constructed, each of which fits the empirical saturation density, the binding energy and the symmetry energy of nuclear matter. They obtained a value of compression modulus at saturation nuclear matter density for symmetric matter that is different from the experimental expected value. So, they allowed a variation in the values of coupling constants arbitrarily so that the theory will give rise to the desired value of compression modulus. The vector field plays no role in determining the value of the effective mass of the nucleon in such an approach.

V. Baron, Cooperstein and Kahana model:

Baron *et al*. [43] gave a phenomenological model for neutron matter equation of state. The form of the nuclear partial pressure is

$$P = \frac{K_0 \rho_0}{9\gamma}[u^\gamma - 1], \tag{2.22}$$

where the baryon density compression factor $u = \rho/\rho_0$, measured with respect to the empirical symmetric nuclear matter saturation density, $\rho_0 = 2.8 \times 10^{14}\ g\ cm^{-3} = 0.16\ fm^{-3}$. Here, K_0 is the nuclear incompressibility at saturation, *i.e.* $u = 1$, while γ is the extremely high-density adiabatic index.

Later, Cooperstein [44] modified the phenomenological equation of state to the following form

$$P = \frac{K_N \rho_0}{9\gamma} u^\gamma, \tag{2.23}$$

$$\varepsilon = \rho_0(m_n + m_1)u + \frac{P}{\gamma - 1}, \tag{2.24}$$

where the integration constant m_1 is given by

$$m_1 = E_{nm} + E_{sym} - \frac{K_N}{9\gamma(\gamma - 1)}, \tag{2.25}$$

with the empirical value $E_{nm} = -16 MeV$ and for definiteness $E_{sym} = 36\ MeV$. Here the factor u^γ is replaced by $u^\gamma - 1$ for nuclear matter saturation. The neutron matter compression modulus is denoted by K_N and m_n is the nucleon mass. The value of K_N is not interpreted as the correct saturation density but is represented as a high density neutron matter parameter. This is connected with K_0 through the density dependence of the symmetry energy. One can find different equations of state by varying the values of K_N and γ in this theory. For particular values of K_N and γ, these phenomenological equations of state fit well the non-relativistic equation of states, such as that of the Bethe and Johnson [17] or Friedman and Pandharipande [26].

VI. Rosenhauer et al. model:

Rosenhauer *et al.* [45] refer two commonly used parameterizations for the hadronic equation of state, namely that of Sierk and Nix [46]

$$\varepsilon_{SN}(n) = \frac{2K}{9}(\sqrt{n/n_0} - 1)^2 \tag{2.26}$$

and the quadratic form [47] is

$$\varepsilon_Q(n) \;=\; \frac{K}{18}(n-n_0)^2/{n_0}^2. \tag{2.27}$$

The symbol $\varepsilon_{SN}(n)$ and $\varepsilon_Q(n)$ represent energy density of the two equations of state, namely, Sierk and Nix as well as quadratic form respectively. Here, n_0 is the normal nuclear matter density and K, the compression modulus characterizing the properties of nuclear matter at densities $n > n_0$. In these two parametric equations, $\varepsilon_{comp}(n)$ (*i.e.*, $\varepsilon_{SN}(n)$ or $\varepsilon_Q(n)$), the large K value indicates the stronger repulsive nature of nucleon-nucleon interaction.

The total energy density is then expressed as

$$\varepsilon(n) \;=\; n[\varepsilon_{comp}(n) + W_0 + m_n + W_{syms}], \tag{2.28}$$

where the binding energy per nucleon at normal nuclear matter density, ($n_0 = 0.145\,fm^{-3}$) is $W_0 = -16\ MeV$, m_n being the rest mass of the neutron and $W_{sym} = 32\ MeV$, the symmetry energy of neutron matter estimated from the liquid drop model. The symmetry energy W_{sym} and the binding energy W_0 determine the properties of matter at saturation within this phenomenological approach. However, $\varepsilon_{comp}(n)$ incorporates all density dependent effects.

VII. Zimanyi and Moszkowski model:

Zimanyi and Moszkowski [8] proposed a model similar to the Walecka model but using the derivative scalar coupling (DSC) for the scalar field. The interesting feature of this model is that the equation of motion of the scalar field becomes non-linear without introducing any extra parameter. This model gives rise to a reasonable value of the effective nucleon mass and satisfactory value for the incompressibility of nuclear matter at saturation density.

Glendenning *et al.* [48] extended the above model to hyperonic matter and applied to compute a number of neutron star properties. In place of the purely derivative coupling of the scalar field to the baryons and vector meson fields of the above (DSC) model, they here coupled it by both Yukawa point and derivative coupling to baryons and both vector fields. This improves the value of the compression modulus and effective nucleon mass at saturation density compared to that obtained in earlier calculation. Also, they include the ρ- meson to account for the asymmetry effect.

Chapter 3

Chiral sigma model

3.1 Introduction

The chiral symmetry is a good hadron symmetry, which ranks only below the isotopic spin symmetry [39]. Due to this reason, it is expected that any theory of dense matter should possess it. If one considers the chiral symmetry in dense matter theory, it should be capable of describing the bulk properties of nuclear matter such as binding energy per nucleon, saturation density, compression modulus and symmetry energy. So far, there has not been any theory possessing chiral symmetry and describing all the nuclear matter properties. In recent years, the importance of the three-body forces in the equation of state at high densities has been emphasized by several authors [49, 50]. This gives theoretical impetus to study the chiral sigma model, because the non-linear terms in the chiral sigma Lagrangian can give rise to the three-body forces.

A chiral Lagrangian using the scalar field (the so-called sigma model) was originally introduced by Gell-Mann and Levy [51] as an example to illustrate chiral symmetry and partial conservation of axial current. The importance of chiral symmetry in the study of

nuclear matter properties was emphasized by Lee and Wick [52]. The non-linear terms of the chiral Lagrangian can provide the three-body forces, important at high densities (Jackson, Rho and Krotscheck [49]; Ainsworth *et al.* [50]), and can be relevant in applications to neutron star structure and supernova collapse dynamics.

The usual theory of pions leads to a theory of nuclear matter that does not possess the empirically desirable saturation property. For this reason, the isoscalar vector field is introduced in the theory via the Higgs mechanism. This way it becomes possible to have a saturating nuclear matter equation of state (Boguta [53]). With the availability of experimental estimate for the incompressibility parameter of nuclear matter (denoted by K), there have been attempts to reproduce the desirable value of K (about $200 - 300$ MeV) using the sigma model. In the 'standard' sigma model, the value of K turns out to be quite large, several times the above-mentioned value for plausible values of the coupling constants involved, and can be reduced only by introducing in the theory, terms due to the scalar field self-interactions and/or vacuum fluctuations with adjustable coefficients.

There have been several earlier papers that have employed the chiral sigma model to obtain the equation of state of high density matter. Glendenning [40] derives the chiral sigma model equation of state with normal nuclear matter saturation and then applies to neutron star structure calculations. He then studies the finite temperature behaviour of this model. A liquid-gas phase transition occurs in this model below $T \sim 23\ MeV$. The equation of motion is derived in the mean field approximation, supplemented by a gauge massless vector meson in interaction with the other hadrons, including baryon resonances. The gauge field of a massless vector mesons is introduced through the covariant derivatives into the chiral sigma model. The linear term of sigma field in the Lagrangian is the

symmetry breaking term by which the pion acquires a finite mass. There are scalar meson and pseudoscalar pion in the theory, in addition to ω- meson, which contribute a repulsive force and carry spin and isospin. Also, he considers this model with dynamical masses but not with the ρ- meson and its isospin symmetry influences. The required parameters in this theory are calculated from the saturation density and the binding energy per nucleon in normal symmetric matter. The value of the compression modulus obtained from this model is too large (650 MeV).

In another paper, Glendenning [41] extends the chiral sigma model to normal nuclear matter and neutron star matter by considering the vacuum renormalisation corrections. The equation of state is derived by including ρ- mesons and hyperons in equilibrium with nucleons and electrons and muons. In this calculation the ω- meson does not have a dynamically generated mass. He has considered two sets of coupling constants corresponding to two compression moduli, a 'stiff' ($K = 300\ MeV$) and 'soft' ($K = 200\ MeV$) equation of states for nuclear matter, that yield the empirical saturation density, the binding as well as the symmetry energy. In both cases, the hyperons influence on the equation of state substantially.

An equation of state based on the chiral sigma model is also considered by Prakash and Ainsworth [42]. They examine the role of the many-body effects provided by the chiral sigma model in the equation of state of symmetric nuclear matter and neutron-rich matter. Previously, Matsui and Serot [54] considered the role of only the nucleon vacuum fluctuation terms at the one loop level. But here, these authors consider both the nucleon as well as the σ meson vacuum fluctuation terms at one loop level in the chiral sigma model. As a result, the meson loop shifts the saturation density and thus incompressibility

increases. In this theory, the isoscalar vector field is not generated dynamically. Its role is an empirical one. They have constructed a family of equation of states, each of which fits the empirical saturation density, the binding energy and the symmetry energy of nuclear matter. They allow variation of the coupling constants arbitrarily in their calculations by fitting the other parameters from nuclear saturation properties, so that it is possible to obtain any desired value of the nuclear matter compression modulus. Finally, they calculate neutron star structure using the neutron matter equation of state for p, n and e systems.

We have developed the $SU(2) \times SU(2)$ chiral sigma model [55] to describe nuclear matter and neutron star matter. We have adopted an approach in which the isoscalar vector field is generated dynamically. Inclusion of such a field is necessary to ensure the saturation property of nuclear matter. The effective mass of the nucleon thus acquires a density dependence on both the scalar as well as the vector fields, and must be obtained self-consistently. We do this using the mean-field theory where all the meson fields are replaced by their uniform, expectation values. To describe the nuclear matter we have two parameters in the theory : (i) the ratio of the coupling constant to the mass of the scalar and (ii) to the isoscalar vector fields. This procedure also gives a relatively high value for K at the saturation density. Although this is an undesirable feature as far as nuclear matter at saturation density is concerned, it need not be viewed as a crucial shortcoming for our purpose here in view of the fact that a fit to K at saturation does not tell us what the slope of the equation of state should be at densities $\geq 4n_s$ (n_s = saturation density) (Prakash and Ainsworth [42]; Horowitz and Serot [56]; Stock [57]; Baym [58]; Ellis, Kapusta and Olive [59]). For neutron star structure, in which we are most interested as an application

of our equation of state, this high density regime plays the most important part. The important feature of our work is that the ω- meson mass and coupling constant are not treated empirically, but in a fully self-consistent manner. To describe neutron star matter, we include the coupling to the isovector ρ- meson, the coupling strength being determined by requiring a fit to the empirical value of the symmetry energy.

3.2 The Model

The Lagrangian for an $SU(2) \times SU(2)$ chiral sigma model that includes (dynamically) an isoscalar vector field (ω_μ) is (we choose $\hbar = 1 = c$) :

$$\begin{aligned} \mathcal{L} = \frac{1}{2}(\partial_\mu \overrightarrow{\pi}.\partial^\mu \overrightarrow{\pi} + \partial_\mu \sigma \partial^\mu \sigma) - \frac{\lambda}{4}(\overrightarrow{\pi}.\overrightarrow{\pi} + \sigma^2 - x_o^2)^2 \\ -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}{g_\omega}^2(\sigma^2 + \overrightarrow{\pi}^2)\omega_\mu\omega^\mu \\ +g_\sigma\bar{\psi}(\sigma + i\gamma_5\overrightarrow{\tau}.\overrightarrow{\pi})\psi + \bar{\psi}(i\gamma_\mu\partial^\mu - g_\omega\gamma_\mu\omega^\mu)\psi, \end{aligned} \tag{3.1}$$

where $F_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, ψ is the nucleon isospin doublet, $\overrightarrow{\pi}$ is the pseudoscalar pion field and σ is the scalar field. The vector field ω_μ couples to the conserved baryonic current $j_\mu = \bar{\psi}\gamma_\mu\psi$. The expectation value $< j_o >$ is identifiable as the nucleon number density, which we denote by ρ_B.

The interactions of the scalar and the pseudoscalar mesons with the vector boson generates a mass for the latter spontaneously by the Higgs mechanism. The masses for the nucleon, the scalar meson and the vector meson are respectively given by

$$M = g_\sigma x_o;$$

$$m_\sigma = \sqrt{2\lambda}x_o;$$

$$m_\omega = g_\omega x_o, \tag{3.2}$$

where x_o is the vacuum expectation value of the sigma field, $\lambda = ({m_\sigma}^2 - {m_\pi}^2)/(2{f_\pi}^2)$ (m_π = pion mass and f_π is the pion decay coupling constant) and g_ω and g_σ are the coupling constants for the vector and scalar fields respectively.

To derive the thermodynamic quantities of the system of degenerate nucleons, characterized by the nucleon number density (ρ_B) or equivalently the Fermi momentum $k_F = (6\pi^2\rho_B/\gamma)^{1/3}$($\gamma$ = nucleon spin degeneracy factor), we need to know the dependence of the meson fields on ρ_B. For this, we resort to the mean–field approximation. This approach has been extensively used to obtain field theoretical equation of state models for high density matter. In this approximation, expected to be valid for degenerate matter at high densities, the mesonic fields are assumed to be uniform (*i.e.*, space-time independent with no quantum fluctuations). For the isoscalar vector field, then

$$\omega_\mu = \omega_o \delta^o_\mu, \tag{3.3}$$

where ω_o is space-time independent but depends on ρ_B and δ^o_μ is the Kronecker delta. The equation of motion for the vector field specifies ω_o :

$$\omega_o = \frac{\rho_B}{g_\omega x^2},$$
$$x = (< \sigma^2 + \overrightarrow{\pi}^2 >)^{1/2}. \tag{3.4}$$

The equation of motion for σ is written for convenience in terms of $y \equiv x/x_o$, and is of the form

$$y(1-y^2) + \frac{c_\sigma c_\omega \gamma^2 k_F^6}{18\pi^4 M^2 y^3} - \frac{c_\sigma y \gamma}{\pi^2}\int_o^{k_F} \frac{dk k^2}{(\overrightarrow{k}^2 + M^{\star 2})^{1/2}} = 0, \tag{3.5}$$

where $M^\star \equiv yM$ is the effective mass of the nucleon and

$$c_\sigma \equiv g_\sigma^2/m_\sigma^2; \qquad c_\omega \equiv g_\omega^2/m_\omega^2. \tag{3.6}$$

We consider here the normal state of high density matter in which there is no pion condensation.

The diagonal components of the conserved total stress tensor corresponding to the Lagrangian (3.1) together with the equation of motion for the fermion field (and a mean field approximation for the meson fields) provide the following identification for the total energy density (ϵ) and pressure (P) of the many-nucleon system (assumed to be a perfect fluid) :

$$\begin{aligned} \varepsilon &= \frac{M^2(1-y^2)^2}{8c_\sigma} + \frac{\gamma^2 c_\omega k_F^6}{72\pi^4 y^2} + \frac{\gamma}{2\pi^2}\int_o^{k_F} dk k^2(\overrightarrow{k}^2 + M^{\star 2})^{1/2} \\ P &= -\frac{M^2(1-y^2)^2}{8c_\sigma} + \frac{\gamma^2 c_\omega k_F^6}{72\pi^4 y^2} + \frac{\gamma}{6\pi^2}\int_o^{k_F} \frac{dk k^4}{(\overrightarrow{k}^2 + M^{\star 2})^{1/2}}. \end{aligned} \tag{3.7}$$

The energy per nucleon is

$$E = \frac{3\pi^2 M^2(1-y^2)^2}{4\gamma c_\sigma k_F^3} + \frac{\gamma c_\omega k_F^3}{12\pi^2 y^2} + \frac{3}{k_F^3}\int_o^{k_F} dk k^2(\overrightarrow{k}^2 + M^{\star 2})^{1/2}. \tag{3.8}$$

For pure neutron matter $\gamma = 2$ and for nuclear matter $\gamma = 4$. A specification of the coupling constants $c_\sigma,\ c_\omega$ now specifies the equation of state.

3.3 Nuclear matter equation of state

For nuclear matter, we fix c_σ and c_ω by fits to two nuclear matter properties : the saturation density (n_s) and the binding energy per particle at $\rho_B = n_s$. For these we

choose the values 0.153 fm^{-3} and -16.3 MeV respectively, as suggested from analysis of experimental data (Möller, Myers, Swiatecki and Treiner [60]). This gives

$$c_\sigma = 6.2033\ fm^2$$
$$c_\omega = 2.9378\ fm^2. \tag{3.9}$$

This leads to a value of $0.78M$ for the effective mass of the nucleon in saturating nuclear matter. The value of K at saturation density that we get is $\sim 700\ MeV$.
Values of the various thermodynamical quantities for nuclear matter and neutron matter for various densities are presented in Table 3.1a and Table 3.1b. The quantity μ appearing in this table is the chemical potential, given by the relationship : $\mu = (P + \varepsilon)/\rho_B$ and ρ is the total mass-energy density, ε/c^2.

In heavy-ion collision experiments, hot hadronic matter is produced at temperatures upto 100 MeV, which contain upto 25% of their energy in nuclear resonances and mesonic degrees of freedom. After making allowance for (model-dependent) corrections for the thermal part of the energy, first estimates of the energy per nucleon of nuclear matter (for $k_BT = 0$) have been made (Stock [57] for a discussion). These estimates by Stock [57] are unlikely to be firm, however, for our purpose we have considered here as guide for comparison with our results. In Fig. 3.1, we present a comparison of available such estimates with the prediction of the equation of state considered by us here. Upto a density of $4n_s$, there is satisfactory agreement between the two.

TABLE 3.1a

EQUATION OF STATE OF DEGENERATE NUCLEAR MATTER
AS GIVEN BY THE PRESENT MODEL

k_F (fm^{-1})	ρ_B fm^{-3}	y	ρ $g\ cm^{-3}$	P $dyn\ cm^{-2}$	E MeV	μ MeV
1.0	0.068	0.91	1.12 E 14	−1.14 E 33	930.98	920.42
1.1	0.090	0.87	1.49 E 14	−1.81 E 33	927.64	915.07
1.2	0.117	0.83	1.92 E 14	−2.41 E 33	924.39	912.95
1.3	0.148	0.79	2.44 E 14	−4.82 E 33	922.52	920.49
1.4	0.185	0.75	3.05 E 14	6.57 E 33	924.41	946.56
1.5	0.228	0.72	3.79 E 14	2.23 E 34	932.77	993.76
1.6	0.277	0.72	4.68 E 14	4.73 E 34	948.91	1055.60
1.7	0.332	0.72	5.75 E 14	8.11 E 34	972.47	1125.00
1.8	0.394	0.74	7.04 E 14	1.23 E 35	1002.30	1197.90
1.9	0.463	0.76	8.57 E 14	1.75 E 35	1037.30	1272.40
2.0	0.540	0.78	1.04 E 15	2.35 E 35	1076.30	1347.60
2.1	0.625	0.80	1.25 E 15	3.05 E 35	1118.40	1423.00
2.2	0.719	0.83	1.49 E 15	3.86 E 35	1163.10	1498.40
2.3	0.822	0.86	1.77 E 15	4.79 E 35	1209.70	1573.80
2.4	0.934	0.88	2.09 E 15	5.85 E 35	1257.90	1649.00
2.5	1.055	0.91	2.46 E 15	7.05 E 35	1307.40	1724.00

TABLE 3.1b

EQUATION OF STATE OF DEGENERATE NEUTRON MATTER
AS GIVEN BY THE PRESENT MODEL

k_F (fm^{-1})	ρ_B fm^{-3}	y	ρ $g\ cm^{-3}$	P $dyn\ cm^{-2}$	E MeV	μ MeV
1.0	0.034	0.95	0.57 E 14	−9.19 E 31	941.03	939.33
1.1	0.045	0.94	0.75 E 14	−2.05 E 32	940.39	937.53
1.2	0.058	0.92	0.98 E 14	−3.76 E 32	939.49	935.47
1.3	0.074	0.90	1.24 E 14	−5.78 E 32	938.41	933.54
1.4	0.093	0.87	1.55 E 14	−6.91 E 32	937.32	932.67
1.5	0.110	0.84	1.90 E 14	−3.56 E 32	936.57	934.62
1.6	0.140	0.80	2.31 E 14	1.26 E 33	936.82	942.49
1.7	0.170	0.77	2.78 E 14	5.66 E 33	939.12	960.40
1.8	0.200	0.75	3.32 E 14	1.46 E 34	944.77	991.16
1.9	0.230	0.73	3.94 E 14	2.93 E 34	954.84	1033.80
2.0	0.270	0.73	4.67 E 14	4.99 E 34	969.73	1084.90
2.1	0.310	0.73	5.52 E 14	7.60 E 34	989.25	1141.00
2.2	0.360	0.74	6.49 E 14	1.08 E 35	1012.90	1199.90
2.3	0.410	0.75	7.62 E 14	1.45 E 35	1040.10	1260.30
2.4	0.470	0.77	8.91 E 14	1.88 E 35	1070.20	1321.50
2.5	0.530	0.79	1.04 E 15	2.37 E 35	1102.70	1383.20

NOTE.- The respective columns stand for Fermi momentum, nucleon number density, the nucleon effective mass factor, the total mass-energy, the pressure, the energy per nucleon and the nucleon chemical potential. The numbers following the letter E represent powers of ten in all the tables.

Fig. 3.1: First generation of estimates, from heavy-ion collision data, of energy per nucleon, E, of nuclear matter plotted against $n_B = \rho_B$ (in units of n_s). The crosses correspond to pion data and the circles to radial energies (Stock [57] for a detailed discussion). The dashed curve corresponds to the present model (nuclear matter).

3.4 Neutron star matter equation of state

At high densities typical of interiors of neutron stars, the neutron chemical potentials will exceed the combined masses of proton and electron. Asymmetric nuclear matter with an admixture of electrons (rather than pure neutron matter) is, therefore, a more likely composition of matter in neutron star interiors. The concentrations of protons and electrons (denoted by n_p and n_e respectively) can be determined using conditions of beta equilibrium ($n \leftrightarrow p + e + \bar{\nu}$) and electrical charge neutrality :

$$\mu_n = \mu_p + \mu_e$$
$$n_p = n_e \tag{3.10}$$

(μ_i = chemical potential of particle species i).

Since nuclear force is known to favour isospin symmetry, and since the symmetry energy arising solely from the Fermi energy is known to be inadequate to account for the empirical value of the symmetry energy ($\simeq 32\ MeV$), we include the interaction due to isospin triplet ρ- meson in Eq. (3.1) for purpose of describing neutron-rich matter. That is, we add the following terms :

$$-\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}m_\rho^2\overrightarrow{\rho_\mu}.\overrightarrow{\rho}^\mu - \frac{1}{2}g_\rho\bar{\psi}(\overrightarrow{\rho}_\mu.\overrightarrow{\tau}\gamma^\mu)\psi \tag{3.11}$$

to the right hand side of Eq. (3.1) in order to describe the asymmetric matter. Here, $\overrightarrow{\rho}_\mu$ stands for the ρ- meson field with mass m_ρ, g_ρ is the coupling strength and

$$G_{\mu\nu} \equiv \partial_\mu\overrightarrow{\rho}_\nu - \partial_\nu\overrightarrow{\rho}_\mu. \tag{3.12}$$

Strictly speaking, the ρ- meson should couple to the total conserved current (Glendenning, Banerjee and Gyulassy [61]). In the above, we have coupled the ρ- meson to the baryons,

which are not the only possible source of isospin current. However, for the ground state equation of state, in the mean-field approximation, only the baryon part of the isospin current will survive (Glendenning [41]).

The equation of motion for $\overrightarrow{\rho}_\mu$, in the mean field approximation where $\overrightarrow{\rho}_\mu$ is replaced by its uniform value ρ_o^3 (here superscript 3 stands for the third component in isospin space), gives

$$m_\rho^2 \rho_o^3 = \frac{1}{2} g_\rho \sum_{B=n,p} < \bar{\psi} \gamma_o \tau^3 \psi >_B, \tag{3.13}$$

where the sum is over neutrons and protons. This gives the following density dependence for the field variable ρ_o^3 :

$$\rho_o^3 = \frac{g_\rho}{2m_\rho^2}(n_p - n_n). \tag{3.14}$$

The symmetric energy coefficient that follows from the semi-empirical nuclear mass formula (that is, the coefficient of the term $(n_p - n_n)^2/(n_p + n_n)^2$ in the mass formula), is:

$$a_{sym} = \frac{c_\rho k_F^3}{12\pi^2} + \frac{k_F^2}{6(k_F^2 + M^{\star 2})^{1/2}}, \tag{3.15}$$

where $c_\rho \equiv g_\rho^2/m_\rho^2$ and $k_F = (6\pi^2 \rho_B/\gamma)^{1/3}$ $(\rho_B \ = \ n_p + n_n)$. We fix the coupling constant c_ρ by requiring that a_{sym} correspond to the empirical value 32 MeV. This gives

$$c_\rho = 4.6617 \ fm^2. \tag{3.16}$$

It is noted that the ρ- meson will contribute a term $= m_\rho^2(\rho_o^3)^2/2$ to the energy density and pressure. Table 3.2 lists the pressure versus the total mass-energy density for the neutron-rich matter in beta equilibrium.

TABLE 3.2

PRESSURE VS DENSITY FOR NEUTRON-RICH MATTER

ρ ($g\ cm^{-3}$)	P ($dyn\ cm^{-2}$)
4.531 E 15	1.619 E 36
3.686 E 15	1.285 E 36
2.695 E 15	8.978 E 35
2.109 E 15	6.719 E 35
1.784 E 15	5.472 E 35
1.504 E 15	4.406 E 35
1.211 E 15	3.296 E 35
1.017 E 15	2.565 E 35
9.116 E 14	2.171 E 35
8.175 E 14	1.822 E 35
7.028 E 14	1.400 E 35
6.059 E 14	1.051 E 35
5.576 E 14	8.813 E 34
5.037 E 14	6.967 E 34
4.561 E 14	5.410 E 34
4.063 E 14	3.887 E 34
3.564 E 14	2.534 E 34
3.018 E 14	1.352 E 34

3.5 Results

Using the chiral sigma model and adopting the approach that the isoscalar vector field, needed to provide saturating binding energy of degenerate nuclear matter be generated dynamically, we have obtained an equation of state of degenerate nuclear and neutron-rich matter at high densities.

The maximum mass of the neutron stars is calculated by integrating the structure equations, which will be discussed in the chapter 7 in detail. A comparison with the maximum mass of (nonrotating) neutron stars predicted by others, using models based on recent field theoretical equations of state is given in Table 3.3. The choice of the equation-of-state models in Table 3.3 is representative, but by no means exhaustive. Included in this comparison are equation of state models due to Alonso and Cabanell [37] and Prakash and Ainsworth [42] which are also based on the sigma model, but differ from our model in the details. The equation of state (II) of Alonso and Cabanell [37], comes from a determination of the free parameters of the linear sigma model with an explicit symmetry-breaking term and is coupled to ω- and ρ- mesons, in a renormalizable way. Prakash and Ainsworth [42] included the sigma meson one-loop contributions, but the isoscalar vector field was not generated dynamically, so that its role is reduced to an empirical one, allowing for arbitrary variations in its coupling constant (thereby making it possible to obtain any desired value of the nuclear matter compression modulus). The vector field plays no role in determining the value of the effective mass of the nucleon in such an approach. A comparison of this equation of state with our model for neutron-rich matter and pure neutron matter is shown in Fig. 3.2. The equation of state of Serot

TABLE 3.3

MAXIMUM MASS NEUTRON STARS FOR FIELD THEORETICAL EQUATION OF STATE MODELS

$Equation\ of\ state\ Reference$	n_s fm^{-3}	K MeV	$M_{max}/M_{\odot}$	R km
Serot [62]	0.193	540	2.54	12.28
Chin [63]	0.193	471	2.10	10.57
Alonso-Cabanell [37] equation of state II	0.172	225	1.94	10.90
Prakash-Ainsworth [42] ($g_\omega^2 = 16.27$)	0.160	225	1.83	9.89
Glendenning [35]	0.153	300	1.79	11.18
Present model [55]	0.153	700	2.59	14.03

NOTE.- n_s and K are nuclear matter saturation density and compression modulus.

[62] is calculated in the mean-field approximation in the Walecka model including $\sigma-$, ω- and ρ- mesons and that of Chin [63] with one-loop corrections in the $\sigma - \omega$ model. The equation of state due to Glendenning [41] is along similar lines, and includes one-loop corrections and also scalar self-interactions (upto quartic order), whose magnitudes are adjusted to reproduce empirical saturation properties. This work takes into account the effect of hyperons in beta equilibrium in addition to electrons and muons.

Table 3.3 implies that the present neutron-rich matter equation of state is comparatively 'stiff' as far as neutron stars are concerned. This is reflected in the value of the maximum mass of neutron stars, which is the largest for the present model. It may be mentioned here that observational evidence in favour of a stiff equation of state comes from the identification by Trümper *et al.* [64] of the 35 day cycle of the pulsating X-ray source Her X-1 as originating in free precession of the rotating neutron star (Pines [65]).

In constructing the neutron star matter equation of state, we have restricted ourselves

Fig. 3.2: Pressure versus total mass-energy density. Curve 1 corresponds to neutron-rich matter in beta equilibrium (including the contribution from $\rho-$ meson exchange) : present model. Curve 2 corresponds to pure neutron matter : present model. Curve 3 (dashed curve) is due to Prakash and Ainsworth [42].

to (n, p, e^-) matter in beta equilibrium. At high densities, hyperons can also appear (Glendenning [35]; Kapusta and Olive [36]); this is expected to reduce the 'stiffness' in the equation of state, and a consequent reduction in the maximum mass of neutron stars. Another point that we have not investigated here is the possible role of the $\rho-$ meson tensor interaction as far as the symmetry energy is concerned.

Chapter 4

Astrophysical application I : Structure and radial oscillation of nonrotating neutron stars

4.1 The Neutron Star Structure

The structure of a neutron star is characterized by its mass and radius. Additional parameters of interest are the moment of inertia and the crust thickness. These are important for the dynamics and transport properties of pulsars.

The space-time for a spherically symmetric gravitating system is described by the Schwarzschild metric

$$ds^2 = e^{\nu}c^2dt^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) - e^{\lambda}dr^2, \tag{4.1}$$

where ν, λ are functions of r only [117]. Corresponding to this space-time metric, the equations that describe the hydrostatic equilibrium of degenerate stars without rotation

in general relativity (*i.e.*, where temperature and convection are not important) can be written as [117] :

$$\frac{dp}{dr} = -\frac{G(\rho + p/c^2)(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)} \tag{4.2}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho, \tag{4.3}$$

where p and ρ are the pressure and total mass energy density and $m(r)$ is the mass contained in a volume of radius r. Given an equation of state p(ρ), Eqs. (7.2) and (7.3), called Tolman-Oppenheimer-Volkoff (TOV) equation, can be numerically integrated, for a given central density, to obtain the radius R and gravitational mass $M_G = m(R)$ of the star.

The moment of inertia I of the rotating neutron star, rotating with angular velocity Ω as seen by a distant observer is given by [118]

$$I = \frac{1}{\Omega}\frac{c^2 R^4}{6G}\Big(\frac{d\bar{\omega}}{dr}\Big)_{r=R}, \tag{4.4}$$

where $\bar{\omega}(r)$, the angular velocity of the star fluid relative to the local inertial frame, is given by

$$\frac{d}{dr}\Big(r^4 j \frac{d\bar{\omega}}{dr}\Big) + 4r^3 \bar{\omega}\frac{dj}{dr} = 0, \tag{4.5}$$

where

$$j(r) = e^{-\nu/2}\Big(1 - 2GM_G/rc^2\Big)^{1/2} \tag{4.6}$$

and satisfies the boundary conditions

$$\left(\frac{d\bar{\omega}}{dr}\right)_{r=R} = 0;\ \bar{\omega}(\infty) = \Omega. \tag{4.7}$$

Here the potential function $\nu(r)$, relating the element of proper time to the element of time at $r = \infty$ is given by

$$\frac{d\nu}{dr} = \frac{2G}{r^2c^2}\frac{(m + 4\pi r^3 P/c^2)}{(1 - 2Gm/rc^2)}. \tag{4.8}$$

The definition (7.4) for I includes the relativistic Lens-Thirring effect to order Ω^2. For neutron star, the relativistic effect are important, and so one gets I as defined by Eq. (7.4) to be in excess of classical definition $(2/5)MR^2$.

For the numerical integration to obtain the structure parameters, it is sufficient to start with an arbitrary value of $\nu(0)$, which is then rescaled to satisfy the surface condition

$$\nu(R) = ln\left(1 - \frac{2GM_G}{Rc^2}\right), \tag{4.9}$$

so that $\nu(\infty) = 0$. Likewise, $\bar{\omega}(0)$ is initially chosen to be an arbitrary constant, and a value of Ω given by

$$\Omega = \bar{\omega}(R) + \frac{1}{3}R\left(\frac{d\bar{\omega}}{dr}\right)_{r=R} \tag{4.10}$$

obtained. A new starting value $\bar{\omega}_{new}(0)$ corresponding to any specified Ω_{new} is given by

$$\bar{\omega}_{new}(0) = \Omega_{new}\bar{\omega}(0)/\Omega. \tag{4.11}$$

To integrate the TOV equations, one needs to know the equation of state $p(\rho)$, for the entire expected density range of neutron star, starting from the high density at the center to the surface densities. The composite equation of state for the entire neutron star density span, was constructed by joining the equation of state of high density neutron-rich

TABLE 7.1

PRESSURE VS DENSITY FOR NEGELE AND VAUTHERIN EQUATION OF STATE

ρ ($g\ cm^{-3}$)	P ($dyn\ cm^{-2}$)
1.586 E 14	8.617 E 32
9.826 E 13	3.807 E 32
6.193 E 13	1.835 E 32
3.767 E 13	8.564 E 31
2.210 E 13	3.789 E 31
1.496 E 13	2.095 E 31
9.611 E 12	1.095 E 31
6.248 E 12	6.184 E 30
3.833 E 12	3.621 E 30
2.202 E 12	2.276 E 30
1.471 E 12	1.694 E 30
9.728 E 11	1.228 E 30
6.610 E 11	8.633 E 29
5.228 E 11	6.741 E 29

matter (that we have discussed in chapter 3) to that given by (a) Negele and Vautherin [119] for the density region $(10^{14} - 5 \times 10^{10})\ g\ cm^{-3}$, (b) Baym, Pethick and Sutherland [120] for the region $(5 \times 10^{10} - 10^{3})\ g\ cm^{-3}$ and (c) Feynman, Metropolis and Teller [121] for $\rho < 10^{3}\ g\ cm^{-3}$. These densities regions are tabulated in Table 7.1–7.3.

For a given equation of state $P(\rho)$, and a given central density $\rho(r=0) = \rho_c$, the Eqs. (7.2-7.3) are integrated numerically with the boundary condition :

$$m(r=0) = 0 \tag{4.12}$$

to give R and M_G. The radius R is defined by the point where $P \simeq 0$, or, equivalently, $\rho = \rho_s$, where ρ_s is the density expected at the neutron star surface (about 7.8 $g\ cm^{-3}$). The total gravitational mass is then given by : $M_G = m(R)$.

TABLE 7.2

PRESSURE VS DENSITY FOR BAYM, PETHICK AND SUTHERLAND EQUATION OF STATE

ρ $(g\ cm^{-3})$	P $(dyn\ cm^{-2})$
5.254 E 10	5.949 E 28
3.313 E 10	3.404 E 28
2.090 E 10	1.938 E 28
1.318 E 10	1.048 E 28
8.312 E 09	5.662 E 27
4.164 E 09	2.356 E 27
1.657 E 09	6.860 E 26
1.045 E 09	4.129 E 26
5.237 E 08	1.629 E 26
2.624 E 08	6.676 E 25
6.589 E 07	1.006 E 25
1.655 E 07	1.435 E 24
6.588 E 06	3.911 E 23
1.044 E 06	2.318 E 22
1.654 E 05	1.151 E 21
2.622 E 04	4.968 E 19
1.044 E 04	9.744 E 18

TABLE 7.3

PRESSURE VS DENSITY FOR FEYNMAN, METROPOLIS AND TELLER EQUATION OF STATE

ρ $(g\ cm^{-3})$	P $(dyn\ cm^{-2})$
2.120 E 03	5.820 E 15
4.500 E 01	1.700 E 14
1.640 E 01	1.400 E 13
1.160 E 01	1.210 E 12
8.150 E 00	1.010 E 11
7.900 E 00	1.010 E 10

We solved [55] the TOV equation numerically using predictor-corrector method, which gives better accuracy compared to existing results of the gravitational mass and the radius. In this method, we use logarithm of pressure and energy density *e.g.*, equation of state to take care of lower energy density and pressure, because at low densities the energy density falls very rapidly at the surface of the star.

Table 7.4 lists neutron star structure parameters as predicted by our equation of state (chiral sigma model) for neutron-rich matter, which has already been discussed in chapter 3. The maximum gravitational mass for stable non-rotating neutron star predicted by our model is 2.59 $M_\odot$ ($M_\odot$ = solar mass). This occurs for a central density of 1.4×10^{15} $g\ cm^{-3}$. The corresponding radius and entire crustal length Δ of the star are 14.03 km and 1 km respectively. The crust length Δ is defined as the distance overwhich the density falls from $\rho = 2.4 \times 10^{14}$ to 7.8 $g\ \ cm^{-3}$ *i.e.*, the surface. Fig. 7.1 and Fig. 7.2 show the plots of mass vs central density and moment of inertia vs mass. The maximum moment of inertia is 4.79×10^{45} $g\ cm^2$.

Observationally, masses of neutron stars are estimated from compact binary systems, one member of which is a pulsar. The most precise estimate comes from the pulsar PSR 1913+16, which gives (1.442 $\pm$0.003) $M_\odot$. A recent compilation of the estimated masses by X-ray pulsars (Nagase [122]) gives the maximum mass (corresponding to Vela X-1 pulsar) to be (1.77 $\pm$0.21) $M_\odot$. Stable neutron star masses predicted by the present equation of state discussed in chapter 3 are thus compatible with the observational estimates. The surface red shift factor provides a probe for neutron star structure, if one presumes that observed $\gamma-$ray bursts are gravitationally red shifted e^+e^- annihilation lines, produced near their surface. The surface red shift ratio (α) defined as

Fig. 7.1: Gravitational mass (M_G) of non-rotating neutron stars versus central density (ρ_c) as predicted by the present model equation of state (neutron-rich matter). The maximum stable mass is 2.59 $M_\odot$. The corresponding central density, $\rho_c = 1.5 \times 10^{15}$ $g\ cm^{-3}$ and the radius is 14.0 km.

Fig. 7.2: Neutron star moment of inertia (I) versus gravitational mass (M_G), as predicted by the present model.

TABLE 7.4

NEUTRON STAR STRUCTURE PARAMETERS FOR THE PRESENT MODEL NEUTRON STAR EQUATION OF STATE

ρ_c $(g\ cm^{-3})$	R (km)	$M/M_\odot$	I $(g\ cm^2)$	α	Δ (km)
3.5 E 14	14.30	1.02	1.31 E 45	0.889	4.06
4.0 E 14	14.73	1.36	2.05 E 45	0.853	3.11
4.5 E 14	14.99	1.64	2.73 E 45	0.822	2.55
5.0 E 14	15.13	1.86	3.30 E 45	0.798	2.18
6.0 E 14	15.18	2.16	4.07 E 45	0.761	1.76
8.0 E 14	14.94	2.45	4.70 E 45	0.719	1.36
1.0 E 15	14.62	2.55	4.79 E 45	0.696	1.18
1.2 E 15	14.30	2.59	4.70 E 45	0.682	1.07
1.4 E 15	14.03	2.59	4.54 E 45	0.674	1.00

NOTE.- The respective columns stand for central density, radius, gravitational mass, moment of inertia calculated for angular velocity $= (GM_G/R^3)^{1/2}$, and the surface red shift ratio α.

$$\alpha = (1 - 2GM_G/Rc^2)^{1/2} \tag{4.13}$$

is expected to be (0.78 ±0.02) on the basis of observed data (Friedman and Pandharipande [26] for a discussion). The present neutron-rich matter equation of state, which has already been discussed, gives for a 1.4 $M_\odot$ neutron star : $R = 14.77\ km$, $I = 2.15 \times 10^{45}\ g\ cm^2$ Δ=3.0 km and the red shift ratio (at the surface) = 0.85. The corresponding central density is $4.06 \times 10^{14}\ g\ cm^{-3}$.

4.2 Neutron stars radial oscillations

Since the original suggestion by Cameron [123] that vibration of neutron star could excite motions, which might have interesting astrophysical applications, there has been a series of investigations into the vibrational properties of neutron stars. The earliest

detailed calculations, carried out by Meltzer and Thorne [124] and Thorne [125] examined the radial as well as nonradial oscillations of neutron stars, using then available equation of state, such as the Harrison-Wakono-Wheeler [126] equation of state. These and other early studies, *e.g.*, Wheeler [127]; Chau [128] and Occhionero [129], indicated that the fundamental mode radial oscillation periods for the neutron stars would typically lie in the vicinity of about 0.4 ms, and that the first few quadrupole oscillations would have periods that are also fractions of a millisecond. Furthermore, these oscillation periods were estimated to be damped by gravitational radiation with damping time scales of the order of one second. Also, there are recent papers on nonrotating, rapidly rotating neutron stars and slowly rotating stars. For example, in 1990, Cutler, Lindblom and Splinter [130] computed the frequencies and damping times due to viscosity and gravitational radiation for the lowest frequency modes of a wide range of nonrotating fully relativistic neutron star models. In a subsequent paper, Cutler and Lindblom [131] reviewed and extended the formalism for computing the oscillation frequency of rapidly rotating neutron stars in the post-Newtonian approximation. Recently, Kojima [132] calculated the rotational shift of normal frequencies in polytropic stellar models in the framework of general relativity. The stellar rotation is assumed to be slow and first-order rotational effects are included to the eigenfrequencies of the nonrotating stars. In this section, we present the calculation of radial oscillation periods of neutron stars, using our equation of state based on the chiral sigma model.

The equation governing infinitesimal radial pulsations of a nonrotating star in general relativity was given by Chandrasekhar [133], and it has the following form :

$$F\frac{d^2\xi}{dr^2} + G\frac{d\xi}{dr} + H\xi = \sigma^2\xi, \tag{4.14}$$

where $\xi(r)$ is the Lagrangian fluid displacement and $c\sigma$ is the characteristic eigenfrequency (c is the velocity of light). The quantities F, G, H depend on the equilibrium profiles of the pressure and density of the star, and are given by

$$F = -e^{-\lambda} e^{\nu} \Gamma p/(p + \rho c^2) \tag{4.15}$$

$$\begin{aligned} G = -e^{-\lambda} e^{\nu} \Big\{ \Gamma p \Big(\frac{1}{2}\frac{d\nu}{dr} + \frac{1}{2}\frac{d\lambda}{dr} + \frac{2}{r} \Big) + \\ p\frac{d\Gamma}{dr} + \Gamma \frac{dp}{dr} \Big\} /(p + \rho c^2) \end{aligned} \tag{4.16}$$

$$H = \frac{e^{-\lambda} e^{\nu}}{p + \rho c^2} \Big\{ \frac{4}{r}\frac{dp}{dr} - \frac{(dp/dr)^2}{p + \rho c^2} - A \Big\} + \frac{8\pi G}{c^4} e^{\nu} p, \tag{4.17}$$

where Γ is the adiabatic index, defined in the general relativistic case as

$$\Gamma = (1 + \rho c^2/p) \frac{dp}{d(\rho c^2)} \tag{4.18}$$

and

$$\begin{aligned} A = \frac{d\lambda}{dr}\frac{\Gamma p}{r} + \frac{2p}{r}\frac{d\Gamma}{dr} + \frac{2\Gamma}{r}\frac{dp}{dr} - \frac{2\Gamma p}{r^2} \\ -\frac{1}{4}\frac{d\nu}{dr}\Big(\frac{d\lambda}{dr}\Gamma p + 2p\frac{d\Gamma}{dr} + 2\Gamma\frac{dp}{dr} - \frac{8\Gamma p}{r} \Big) \\ -\frac{1}{2}\Gamma p \Big(\frac{d\nu}{dr} \Big)^2 - \frac{1}{2}\Gamma p \frac{d^2\nu}{dr^2}. \end{aligned} \tag{4.19}$$

The boundary conditions to solve the pulsation equation (7.14) are

$$\xi(r = 0) = 0 \tag{4.20}$$

$$\delta p(r = R) = -\xi \frac{dp}{dr} - \Gamma p \frac{e^{\nu/2}}{r^2} \frac{\partial}{\partial r}(r^2 e^{-\nu/2} \xi)|_{r=R} = 0 \tag{4.21}$$

Here Γ is the adiabatic index. Since p vanishes at $r = R$, it is generally sufficient to demand

$$\xi \;\; finite \;\; at \;\; r = R. \tag{4.22}$$

Equation (7.14), subject to the boundary conditions Eq. (7.20) and (7.21), is a Sturm - Liouville eigen value equation for σ^2.

The following results follow from the theory of such equations :

1. The eigen values σ^2 are all real.

2. The eigen values from an infinite discrete sequence

$$\sigma_o^2 < \sigma_1^2 < \ldots < \sigma_n^2 < \ldots\ldots, \tag{4.23}$$

3. The eigen function ξ_o corresponding to σ_o^2 has no nodes (fundamental mode) in the interval $0 < r < R$; more generally, ξ_n has nodes in this interval.

4. The ξ_n are orthogonal with weight function ωr^2 :

$$\int_0^R \xi_n \xi_m \omega r^2 dr = 0, \;\; m \neq n.$$

5. The ξ_n form a complete set for the expansion of any function satisfying the boundary condition Eqs. (7.20) and (7.22).

An important consequence of item (2) is the following :

If the fundamental radial mode of a star is stable ($\sigma_o^2 > 0$), then all the radial modes are stable. Conversely, if the star is radially unstable, the fastest growing instability will be via the fundamental mode (σ_o^2 more negative than all other σ_n^2).

We solved [134] Eq. (7.14) for the eigenvalue σ by writing the differential equation as a set of difference equations. The equations were cast in tridiagonal form and the eigenvalues were found by using the EISPACK routine. This routine finds the eigenvalues of a symmetric tridiagonal matrix by the implicit QL method.

Results for the oscillations of neutron star corresponding to chiral sigma equation of state are illustrated in Fig. 7.3. The plot in Fig. 7.3 are for the oscillation time period ($= 2\pi/c\sigma$) versus the gravitational mass M_G. The fundamental mode and the first four harmonics are considered. The period is an increasing function of M_G, the rate of increase being progressively less for higher oscillation modes. The fundamental mode oscillation periods for neutron star are found to have the following range of values : (0.35 - 0.50) *milliseconds*. For higher modes, the periods are ≤ 0.2 *milliseconds*. We shall compare our results with those for a different choice of the equation of state, namely, that given by Wiringa *et al.* [11]. We also perform a similar calculation for strange quark stars, using a realistic equations of state.

4.3 Radial oscillations of quark stars

There are strong reason for believing that the hadrons are composed of quarks, and the idea of quark stars has already existed for about twenty years [135, 136]. Calculations of the possible phase transition from baryon matter to quark matter in models of cold, compact stars have been performed by several groups ([95, 137, 96, 138]), but the results are not conclusive concerning the existence of quark matter inside neutron stars. In 1984, it was suggested that strange matter *i.e.*, quark matter with strangeness per baryon of

Fig. 7.3: Periods of radial pulsations as functions of the gravitational mass for our equation of state (chiral sigma model). The labels 1, 2, 3, 4, 5 correspond respectively to the fundamental and the first four harmonics.

order unity, may be the true ground state [99]. The properties of strange matter at zero pressure were subsequently examined, and it was found that strange matter can indeed be stable for a wide range of parameters in the strong interaction calculations [97]. Details of the extension to finite pressure and the so-called strange stars are given in [98, 139]. The problem of the existence of strange stars is, however, still unresolved ([100]).

An important question is how can one possibly distinguish between quark stars and neutron stars. It has been suggested to use measurements of the surface gravitational red shift z (Schwarzschild), since different equation of state gives different results for $z(M_G)$, M being the star mass [140, 141]. The region of allowed high density equation of state may be narrowed further by the observations of pulsar periods [142, 143]. Given a sub-millisecond pulsar, we may argue that the ability of such a fast rotating star to avoid rotational break-up induces severe restrictions and a conventional neutron star will not be able to resist the large centrifugal forces. The problem of rapid rotation of compact stars receives much attention [144, 145, 146, 147], and although no sub-millisecond pulsar is seen among the about 500 pulsars observed so far, further observations may well reveal such an object. Also, among the criteria suggested for distinguishing quark star from neutron star are the neutrino cooling rate [148, 149, 150], transport properties such as bulk viscosity [151, 152], and sub millisecond period rotation rates [145].

These so-called strange stars have rather different mass-radius relationship [139] than neutron stars, but for stars of mass = 1.4 $M_\odot$, the structure parameters of quark stars are very similar to those of neutron stars. Since pulsars are believed to be (rotating) neutron stars, and since available binary pulsar data suggest their masses to be close to 1.4 $M_\odot$, it has been conjectured [99] that at least some pulsars could be quark stars.

Recently, Haensel *et al*. [153] have emphasized that pulsation properties of a neutron star can yield information about the interior composition, namely, whether the interior has undergone a phase transition to quarks. The main idea is to know the damping times, which will be modified if there is a quark matter core. In their study, Haensel *et al*. [153] used polytropic model for the equation of state for nuclear matter as well as quark matter, and the Newtonian pulsation equation to calculate the eigenfrequencies. The strange quark mass and the quark interactions are important for the structure of quark stars [99]. This suggests that the equation of state of strange quark matter will have a role to play in determining the pulsation features of quark stars. Clearly, for a more exact understanding of the vibrational properties of quark stars, use of realistic equation of state for quark matter, and the general relativistic pulsation equation, are desirable. Cutler *et al*. [130] have calculated the frequencies and damping times of radial pulsations of some quark star configurations, using the general relativistic pulsation equation, but for quark matter, they adopted the MIT bag model in its simplest form, namely, non-interacting and massless quarks. The purpose of this work [134] is to calculate the range of eigenfrequencies of radial pulsations of stable quark stars (using the general relativistic pulsation equation) and to investigate the sensitivity of the eigenfrequencies on the equation of state.

The equation of state used by us incorporates short-range quark-gluon interactions perturbatively to second order in the coupling constant α_c. The long-range interactions are taken into account phenomenologically by the bag pressure term B. We incorporate the density dependence of α_c by solving the Gell-Mann-Low equation for the screened charge. The parameters involved are the strange quark mass m_s, B and, the renormalization point μ_o, obtained by demanding that the bulk strange matter be stable at zero temperature

and pressure, with energy per baryon less than the lowest energy per baryon found in nuclear matter. For completeness, we also do the calculations for the MIT bag model.

At high baryonic densities, bulk strange matter is in an overall colour singlet state, and can be treated as a relativistic Fermi gas interacting perturbatively. The quark confinement property is being simulated by the phenomenological bag model constant B. Chemical equilibrium between the three quark flavours and electrical charge neutrality allow us to calculate the equation of state from the thermodynamic potential of the system as a function of the quark masses, the bag pressure term B and the renormalization point μ_o. To second order in α_c, and assuming u and d quarks to be massless, the thermodynamic potential is given by [154] :

$$\Omega = \Omega_u + \Omega_d + \Omega_s + \Omega_{int.} + \Omega_e, \tag{4.24}$$

where Ω_i $(i = u,\ d,\ s,\ e)$ represents the contributions of $u,\ d,\ s$ quarks and electrons and Ω_{int} is the contribution due to interference between u and d quarks and is of order α_c^2 : Expressions for Ω_i and Ω_{int} are already given in chapter 5.

The total energy density and the external pressure of the system are given by

$$\epsilon = \Omega + B + \sum_i \mu_i n_i \tag{4.25}$$

and

$$p = -\Omega - B, \tag{4.26}$$

where n_i is the number density of the i-th particle species. For specific choices of the parameters of the theory (namely, m_s, B and μ_o), the equation of state is now obtained by calculating ϵ and p for a given value of μ :

$$\mu \equiv \mu_d = \mu_s = \mu_u + \mu_e, \tag{4.27}$$

by solving for μ_e from the condition that the total electric charge of the system is zero.

There is an unphysical dependence of the equation of state on the renormalization point μ_o, which, in principle, should not affect the calculations of physical observables if the calculations are performed to all orders in α_c [139, 103]. In practice, the calculations are done perturbatively and, therefore, in order to minimize the dependence on μ_o the renormalization point should be chosen to be close to the natural energy scale, which could be either $\mu_o \simeq B^{1/4}$ or the average kinetic energy of quarks in the bag, in which case, $\mu_o \simeq 313\ MeV$. In the present study, our choice of μ_o is dictated by the requirement that stable strange matter occurs at zero temperature and pressure with a positive baryon electric charge [154]. This leads to the following representative choice of the parameter values:

Equation of state model 1 : $B = 56\ MeV\ fm^{-3}$; $\mathrm{m}_s = 150\ MeV$; $\mu_o = 150\ MeV$.

Equation of state model 2 : $B = 67\ MeV\ \mathrm{fm}^{-3}$' $\mathrm{m}_s = 150\ MeV$; $\alpha_c = 0$.

Model 2 corresponds to no quark interactions, but a non-zero mass for the strange quark. In the limit, $\mathrm{m}_s \to 0$ and $\alpha_c \to 0$, the equation of state has the analytical form

$$p = \frac{1}{3}(\epsilon - 4B), \tag{4.28}$$

where ϵ is the total energy density. Eq. (7.28) is the MIT bag model. It is independent of the number of quark flavours.

Numerical values of pressure p and total mass-energy density $\rho = \varepsilon/c^2$ for the quark matter equation of state models used here are listed in Table 7.5. For the sake of comparison, we have included in this table, the equation of state corresponding to non-interacting,

TABLE 7.5
EQUATIONS OF STATE FOR DEGENERATE STRANGE QUARK MATTER

$\rho(10^{14}\ g\ cm^{-3})$	$P(10^{36}\ dyn\ cm^{-2})$		
	model 1	*model* 2	$MIT\ bag\ (B = 56\ MeV\ fm^{-3})$
6.0	4.44	2.23	6.01
8.0	10.13	7.83	12.00
10.0	15.88	13.49	17.99
12.0	21.63	19.17	23.99
14.0	27.41	24.88	29.98
16.0	33.20	30.16	35.97
18.0	39.00	36.36	41.96
20.0	44.82	42.12	47.95
22.0	50.64	47.89	53.95
24.0	56.47	53.67	59.94
26.0	62.30	59.46	65.93
28.0	68.14	65.26	71.92
30.0	73.98	71.06	77.91
32.0	79.83	76.87	83.90
36.0	91.53	88.51	95.89
40.0	100.32	100.16	107.87
50.0	132.58	129.36	137.83

massless quarks as given by the simple MIT bag model with $B = 56\ MeV\ fm^{-3}$. Among these equation of state, the bag model is 'stiffest' followed by models 1 and 2. Equilibrium configurations of strange quark stars, corresponding to the above equation of state, are presented in Table 7.6, which lists the gravitational mass M_G, radius R, the surface red shift z, given by

$$z = (1 - 2GM/c^2R)^{-1/2} - 1 \tag{4.29}$$

and the period P_o corresponding to fundamental frequency Ω_o defined as [130] :

$$\Omega_o = (3GM/4R^3)^{1/2} \tag{4.30}$$

TABLE 7.6

EQUILIBRIUM STRANGE QUARK STAR MODELS

$Equation\ of\ state$	$\rho_c(10^{14}\ g\ cm^{-3})$	$M/M_\odot$	$R(km)$	$Surface\ red\ shift\ (z)$	$P_o(ms)$
Model 1	24.0	1.958	10.55	0.487	0.488
	20.0	1.967	10.78	0.472	0.503
	16.0	1.951	11.02	0.448	0.522
	12.0	1.864	11.22	0.401	0.548
	8.0	1.521	11.02	0.299	0.591
	6.0	0.997	9.93	0.192	0.624
	5.0	0.485	7.99	0.104	0.646
Model 2	24.0	1.863	10.09	0.483	0.468
	20.0	1.862	10.29	0.465	0.482
	16.0	1.829	10.49	0.435	0.500
	12.0	1.710	10.62	0.381	0.527
	8.0	1.281	10.14	0.263	0.568
	6.0	0.645	8.37	0.138	0.600
	5.0	0.092	4.48	0.032	0.622
MIT Bag	24.0	2.021	10.81	0.493	0.500
model	20.0	2.033	11.04	0.480	0.514
$(B = 56$	16.0	2.023	11.29	0.450	0.533
MeV fm$^{-3})$	12.0	1.947	11.52	0.410	0.558
	8.0	1.635	11.41	0.310	0.604
	6.0	1.150	10.52	0.210	0.636
	5.0	0.666	8.98	0.130	0.659

as functions of the central density ρ_c of the star.

We calculated the eigenvalue σ by solving the Eq. (7.14) as same way as discussed in the previous section.

Results for the oscillations of quark stars corresponding to equation of state models 1 and 2 are illustrated in Fig. 7.4. To compare, we have included in Fig. 7.4 the results for quark stars corresponding to (a) the simple MIT bag equation of state (non-interacting, massless quarks and $B = 56\ MeV\ \text{fm}^{-3}$) and (b) neutron stars corresponding to a recently given neutron matter equation of state [11]. The plots in Fig. 7.4 are for the oscillation time period ($= 2\pi/c\sigma$) versus the gravitational mass M_G. The fundamental mode and the first four harmonics are considered. The period is an increasing function of M, the rate of increase being progressively less for higher oscillation modes. The fundamental mode oscillation periods for quark stars are found to have the following range of values:

MIT bag model : (0.14 - 0.32) *milliseconds*

Equation of state model 1 : (0.10 - 0.27) *milliseconds*

Equation of state model 2 : (0.06 - 0.30) *milliseconds*

For neutron stars (model UV14 + UVII, ref.[11]), we find that the range of periods for l = 0 mode is $(0.25 - 0.4)$ *milliseconds*, which is slightly less than the neutron star based on chiral sigma model equation of state, presented in previous section. For higher modes, the periods are ≤ 0.1 *milliseconds*, similar to the case of quark stars but less than the chiral sigma model values. So, the oscillation periods for neutron star based on chiral sigma model equation of state is different from quark stars, which is insignificant.

Inclusion of strange quark mass and the quark interactions make the equation of state a little 'softer' as compared to the simple MIT bag equation of state (Table 7.5). This is

Fig. 7.4: Periods of radial pulsations as functions of the gravitational mass. The top two and bottom left boxes correspond to strange quark stars. The bottom right box is for stable neutron stars corresponding to beta-stable neutron matter, model UV14 + UVII, ref. [11]. The labels 1, 2, 3, 4, 5 correspond respectively to the fundamental and the first four harmonics.

reflected in the value of the maximum mass of the strange quark star (Table 7.6). For the pulsation of quark stars, this gives, for l=0 mode eigenfrequencies, values as low as 0.06 *milliseconds*. The main conclusion that emerges from our study, therefore, is that use of realistic equation of state can be important in deciding the range of eigenfrequencies, at least for the fundamental mode of radial pulsation. The results presented here thus form an improved first step of calculations on the lines presented by Haensel *et al*. [153], whose numerical conclusions are expected to get altered.

Since we considered the vibrations of neutron stars and quark stars, it is important to study the time scale for damping of the vibrations. Regarding the time scale of damping of the vibration, here we don't calculate exactly, but the damping time is approximately same as Ref. [155], considered by Madsen, because, the oscillation time is taken to be $10^{-3}s$, which is typical for the fundamental mode in our case. The discussion by Madsen [155] was based on rather crude estimates.

Chapter 5

Astrophysical application II : Cooling of neutron stars with quark core

5.1 Introduction

When a neutron star is formed in the collapse of a stellar core, it rapidly cools down by neutrino radiation. The interior temperature drops to about less than 10^{10} K within minutes and to about 10^9 K within one year. Neutrino emission dominates the subsequent cooling of the neutron star, until the interior temperature falls to about 10^8 K, with a corresponding surface temperature of about 10^6 K. Thus the photoemission also begin to play an important role.

The cooling curves (observed temperature as a function of time) depend on a number of interesting aspects of the physics of neutron star. It turns out that the equation of state as well as the mass of the neutron stars do not influence the cooling in a sensitive manner, but the possible existence of a superfluid state of the nucleons plays some role, and the

existence of a pion condensate or a quark phase in the central region would have dramatic effects. For conventional cooling scenarios, neutrino emission dominates the cooling for about 10^5 years.

The theoretical cooling models [156, 157] have been refined [158, 159] in recent years by a number of authors. The search for thermal radiation of pulsars has so far in most cases has led only to upper bounds for the surface temperature, still interesting comparisons between theory and observations can be made.

The so-called standard model of neutron star cooling is based upon neutrino emission from the interior that is dominated by the *modified URCA process* [160];

$$(n,p) + p + e^- \leftrightarrow (n,p) + n + \nu_e, \tag{5.1}$$

$$(n,p) + n \leftrightarrow (n,p) + p + e^- + \bar{\nu}_e. \tag{5.2}$$

The *direct URCA process*

$$n \rightarrow p + e^- + \bar{\nu}_e, \tag{5.3}$$

$$p + e^- \rightarrow n + \nu_e, \tag{5.4}$$

is not usually considered because it is strongly suppressed in degenerate matter because of the requirement of energy and momentum conservation. Since this process has recently been revived [161], we repeat the simple argument. The fermions $n,\ p,\ e^-$ participating in the process have energies lying within T of the Fermi surface. By energy conservation, the neutrino and antineutrino energies are then also $\sim T$. But the Fermi momenta of the

electrons and protons are small compared to the neutron Fermi momentum and thus the processes (8.2) are strongly suppressed by momentum conservation.

This is no longer a case when an additional neutron, which can absorb energy and momentum, takes part in the process, as in (8.1), (8.2). A pion condensate would have the same effect as that of a spectator neutron.

It has recently been argued [161] that the proton concentration in a neutron star might be so high that the momentum conservation, namely

$$p_f(p) + p_f(e) > p_f(n), \tag{5.5}$$

might be satisfied. For an n, p, e mixture we have $n_p = n_e$ and thus the condition becomes

$$n_n \leq 8n_p. \tag{5.6}$$

The proton fraction $x = n_p/n$, where $n = (n_n + n_p)$ is the total baryon density, is then given by

$$x \geq \frac{1}{9} \simeq 11.1\% \tag{5.7}$$

If the electron chemical potential exceeds the muon rest mass $m_\mu = 105.7\ MeV$, muon will also be present in dense matter, and this will increase the threshold proton concentration. If $\mu_e \gg m_\mu$, the threshold proton concentration is $\simeq 0.148$; for smaller values of μ_e, the threshold concentration lies between $\frac{1}{9}$ and 0.148. At densities typical of the central regions of neutron stars, the calculated proton concentration of matter is very sensitive to the choice of physical model, and in reality it might exceed the threshold value as Lattimer

et al. [161] discuss. Estimates of proton fraction as a function of baryon density for a number of different equation of states indicate that, (a) the estimated proton concentrations depend sensitively on the assumptions made about the microscopic interactions, which are poorly known, and (b) it is quite possible that the proton concentrations are large enough to allow the direct Urca process to occur. As calculations by Wiringa, Fiks and Fabrocini [11] demonstrate, the form of the three-body interaction, especially its isospin dependence has a large influence on the proton fraction.

Let us now estimate the rate at which antineutrino energy emitted per unit volume in the reaction given by (8.3). This may be done by using Fermi's "golden rule". Neglecting for the moment the effects of possible superfluid of neutrons and superconductivity of protons, one finds

$$\dot{E}_\beta = \frac{2\pi}{\hbar} 2 \sum_i G^2 cos^2\theta_C (1 + 3g_A^2) \times n_1(1 - n_2)(1 - n_3) \times \varepsilon_4 \delta^4(p_1 - p_2 - p_3 - p_4), \tag{5.8}$$

where n_i is the Fermi function and the subscript i=1 to 4 refer to the neutron, proton, electron, and antineutrino respectively. The p_i are four-momenta, and ε_4 is the antineutrino energy. The sum over states is to be performed only over possible three momenta p_i in unit volume, and prefactor 2 takes into account the initial spin states of the neutron. The beta-decay matrix element squared, after summing over spins of final particles and averaging over angles, is obtained as $G^2 cos^2\theta_C(1+3g_A^2)$, where $G = 1.436 \times 10^{-49}$ erg cm^3 is the weak-coupling constant, θ_C is the Cabibbo angle, and $g_A = -1.261$ being the axial vector coupling constant. Final electron and proton states must be vacant if the reaction

is to occur, and this accounts for the blocking factors $1 - n_2$ and $1 - n_3$. The electron-capture process (8.4) gives the same energy loss rate as process (8.3), but in neutrinos, and therefore the total luminosity per unit volume of the Urca process is twice of Eq. (8.8). The integrals may be calculated straightforwardly, since the neutrons, protons, and electrons are very degenerate. One thus finds (Boltzmann's constant $k_B = 1$)

$$\dot{E}_{Urca} = \frac{457\pi}{10080} \frac{G^2 cos^2\theta_C (1 + 3g_A^2)}{\hbar^{10} c^5} m_n m_p \mu_e (T)^6 \Theta_t. \tag{5.9}$$

Here Θ_t is the threshold factor $\Theta(p_e + p_p - p_n)$, which is $+1$ if the argument exceeds 0, and is 0 otherwise.

Particle interactions change this result in a number of ways. First, the neutron and proton densities of states are determined by effective mass rather than bare masses. Second, the effective weak-interaction matrix elements can be modified by the medium. These effects are expected to reduce the luminosity, but probably by less than a factor of 10.

The temperature dependence of the direct Urca emissivity may easily be understood from phase-space considerations. The neutrino or antineutrino momentum is $\sim T$, and thus the phase space available in final states might be a three-dimensional sphere of this radius, whose volume is proportional to $(T)^3$. The participating neutrinos, protons, and electrons are degenerate. Therefore, for the reaction to occur, they must have energies that lie within $\sim T$ of the energies at the Fermi surfaces, and thus each degenerate particle contributes a factor $\sim T$.

Yet another possibility for the state of matter at high densities is quark matter, in which quarks can move around essentially as free particles, rather than being bound together as colour singlet entities, such as nucleons and pions. Neutrino emission from

such a system was considered by Iwamoto [148]. The basic processes are the quark analogue of the nucleon direct Urca processes (8.3) and (8.4), *i.e.*,

$$d \rightarrow u + e^- + \bar{\nu}_e \tag{5.10}$$

and

$$u + e^- \rightarrow d + \nu_e. \tag{5.11}$$

The condition for beta equilibrium is

$$\mu_d = \mu_u + \mu_e. \tag{5.12}$$

If quarks and electrons are treated as massless noninteracting particles, this condition is

$$p_f(d) = p_f(u) + p_f(e), \tag{5.13}$$

which is identical to the condition for it to be just possible to conserve momentum for excitations near the respective Fermi surfaces. At threshold the momenta of the u quark, the d quark, and the electron must be collinear, but, as Iwamoto pointed out, the weak-interaction matrix element for this case vanishes. However, if quark-quark interactions are taken into account, the direct Urca process for quarks and electrons that are not collinear will be kinematically allowed. To illustrate this effect, consider the case in which interactions may be treated perturbatively. To first order in the QCD coupling constant α, the quark chemical potentials are given by

$$\mu_i = [1 + \frac{8}{3\pi}\alpha]p_f(i), \;\; i = u, \;\; d, \tag{5.14}$$

while the electron chemical potential is unchanged. Since α is positive, it is easy to see that the conditions for beta equilibrium [Eq. (8.12)] and momentum conservation may be satisfied simultaneously. Angles characterizing deviations from collinearity are typically of order $\alpha^{1/2}$. The calculation of the neutrino and antineutrino emission rates proceeds in essentially the same way as for the nucleon process, and the overall results for the luminosity is

$$\dot{E}_q = \frac{914}{315}\frac{G^2 cos^2\theta_C}{\hbar^{10}c^7}\alpha p_f(d)p_f(u)\mu_e(T)^6. \tag{5.15}$$

This has a form similar to the nucleon Urca rate (8.9), but there are some significant differences. First, there is a factor α, which reflects the fact mentioned above that the weak-interaction matrix element vanishes for collinear relativistic particles, whereas for non-relativistic nucleons the corresponding matrix element is essentially independent of angle. The second difference is that the quantities $p_f(u)$ and $p_f(d)$ take the place of the nucleon masses. Third, the numerical coefficient is different because for quarks the angular dependence of the matrix element is important. However, since $p_f(u)$ and $p_f(d)$ are expected to be less than m_n, and α is less than or of the order of unity, the neutrino luminosity from quark matter is expected to be rather less than the characteristic rate for the nucleon process. However, it is important to note that for quark matter the electron fraction is uncertain. For instance, if u, d, and s quarks may be treated as massless and free, the electron fraction vanishes identically. Detailed estimates of the composition of quark matter for various models are given by Duncan, Shapiro and Wasserman [149] and Alcock, Farhi and Olinto [139]. So far, we have assumed the quark to be massless. While this is a good approximation for u and d quarks, but poor for s quarks, which can

participate in Urca process even in the absence of strong interactions. However, detailed calculations show that the energy-loss rate from processes in which s quarks participate is less than that from the processes where for u and d quarks are involved (Iwamoto [148]).

5.2 Equilibrium neutrino emissivity of quark matter

Iwamoto [148] has derived the formula for ϵ using apparently reasonable approximations and this formula has been widely used [148, 139, 103, 150] to calculate ϵ for two and three flavour quark matter. According to his formula, ϵ is proportional to baryon density n_B, strong coupling constant α_c and sixth power of temperature T for d quark decay. For s quark decay, T dependence of ϵ is same as that for d decay. Furthermore, his results imply that electron and quark masses have negligible effect on ϵ and s quark decay (in case of three flavour quark matter) which play a rather insignificant role.

In this section, we describe [162] an exact numerical calculation of ϵ and a comparison of our results with the Iwamoto formula. Our results show that the Iwamoto formula overestimates ϵ by orders of magnitude when $p_f(u)+p_f(e)-p_f(d(s))$ is comparable with the temperature. For reasonable values of α_c and baryon densities, this quantity is much larger than the expected temperatures of neutron stars ($\sim$ few 10ths of MeV) for two flavour quark matter, but is comparable with temperature for three flavour quark matter. The neutrinos are emitted from the quark matter through reactions

$$d \rightarrow u + e^- + \bar{\nu}_e$$

$$u + e^- \rightarrow d + \nu_e$$

$$s \rightarrow u + e^- + \bar{\nu}_e$$

$$u + e^- \rightarrow s + \nu_e. \tag{5.16}$$

The equilibrium constitution of the quark matter is determined by its baryon density n_B, charge neutrality conditions and weak interactions given in Eq. (8.16). Thus, for two flavour quark matter,

$$\mu_d = \mu_u + \mu_e (\mu_{\nu_e} = \mu_{\bar{\nu}_e} = 0)$$

$$2n_u - n_d - 3n_e = 0$$

$$n_B = (n_u + n_d)/3 \tag{5.17}$$

and for three flavour quark matter

$$\mu_d = \mu_u + \mu_e (\mu_{\nu_e} = \mu_{\bar{\nu}_e} = 0)$$

$$\mu_d = \mu_s$$

$$2n_u - n_d - n_s - 3n_e = 0$$

$$n_B = (n_u + n_d + n_s)/3. \tag{5.18}$$

The number density of species i is $n_i = g.p_f^3(i)/(6\pi^2)$ with the degeneracy factor g_i being two for electron and six for quarks. For electrons $\mu_e = \sqrt{p_f^2(e) + m_e^2}$ and for quarks we use [163]

$$\mu_q = [\frac{\eta}{x} + \frac{8\alpha_c}{3\pi}(1 - \frac{3}{x\eta} ln(x + \eta))]p_f, \tag{5.19}$$

where $x \equiv p_f(q)/m_q$ and $\eta \equiv \sqrt{1 + x^2}$, m_q being the quark mass. For massless quarks, Eq. (8.19) reduces to

$$\mu_q = (1 + \frac{8\alpha_c}{3\pi})p_f(q). \tag{5.20}$$

The neutrino emissivity ϵ for reactions involving $d(s)$ quarks is calculated by using the reactions in Eq. (8.16). In terms of the reaction rates of these equations, we get ($\hbar = c = 1$),

$$\epsilon_{d(s)} = A_{d(s)} \int d^3p_{d(s)} d^3p_u d^3p_e d^3p_\nu \frac{(p_{d(s)}.p_\nu)(p_u.p_e)}{E_u E_{d(s)} E_e}$$
$$\times \delta^4(p_{d(s)} - p_u - p_e - p_\nu) n(\vec{p}_{d(s)})[1 - n(\vec{p}_u)][1 - n(\vec{p}_e)], \tag{5.21}$$

where $p_i = (E_i,\ \vec{p_i})$ are the four momenta of the particles, $n(\vec{p_i}) = \frac{1}{e^{\beta(E_i - \mu_i)}+1}$ are the Fermi distribution functions and

$$A_d = \frac{24G^2 \cos^2\theta_c}{(2\pi)^8} \tag{5.22}$$
$$A_s = \frac{24G^2 \sin^2\theta_c}{(2\pi)^8}. \tag{5.23}$$

For degenerate particles, ($\beta p_f(i) \gg 1$), Iwamoto has evaluated the integrals in Eq. (8.21) using certain reasonable approximations and obtained the simple expressions for ϵ_d and ϵ_s as given below [148],

$$\epsilon_d = \frac{914}{315} G^2 \cos^2\theta_c \alpha_c p_f(d) p_f(u) p_f(e) T^6$$
$$\epsilon_s = \frac{457\pi}{840} G^2 \sin^2\theta_c \alpha_c \mu_s p_f(u) p_f(e) T^6. \tag{5.24}$$

The approximations involved in obtaining these formulas are

1. neglect of neutrino momentum in momentum conservation,

2. replacing the matrix elements by some angle averaged value, and

3. decoupling momentum and angle integrals.

The expressions for neutrino emissivity as obtained by Iwamoto have been used widely. The temperature dependence of emissivity as obtained by Iwamoto has a physical explanation. Each degenerate fermion gives one power of T from the phase space integral ($d^3p_i \rightarrow p_f(i)^2 dE_i \propto T$). Thus one gets T^3 from quarks and electrons. Phase space integral for the neutrino gives $d^3p_\nu \propto (E_\nu^2)dE_\nu \propto T^3$. Energy conserving $\delta-$function gives one T^{-1} which is cancelled by one $E_\nu \propto T$ factor coming from matrix element. So finally, one gets $\epsilon \propto T^6$. This argument, however, ignores the fact that $\Delta p_d \ (\Delta p_s) = p_f(u) + p_f(e) - p_f(d) \ (p_f(s))$, which is related to the angle between $\vec{p}_d$, $\vec{p}_u$ and $\vec{p}_e$ could be small and comparable to T. We shall demonstrate below that precisely in this region that the Iwamoto formula fails.

Before discussing the causes of the shortcoming of Iwamoto formula, let us first compare our results with the Iwamoto formula and try to find out the specific cases where the deviation is more pronounced. In Figs.8.1-8.3, we have plotted ϵ vs T for two-flavour d decay, three-flavour d decay and s decay respectively. For two-flavour d decay our results (ϵ_d) are in good agreement with the emissivity calculated using Iwamoto formula (ϵ_{dI}). In Fig. 8.1 curves (a) and (b) are ϵ_{dI} and ϵ_d respectively, for $\alpha_c = 0.1$ and $n_B = 0.4$. (c) and (d) corresponds to the same but for α_c= 0.1 and n_B= 1.4. It is evident from the figure that agreement of Iwamoto results with our calculation is better for higher densities and lower temperatures. Also ϵ_d is consistently smaller than ϵ_{dI}, the Iwamoto result, in the range of temperatures considered. Corresponding Fermi momenta of quarks and electrons are given in Table 8.1. It is to be noted that all the momenta are much larger than the temperature.

Fig. 8.1: Two flavour d-decay for $\alpha_c = 0.1$; (a) Iwamoto results for $n_B = 0.4 fm^{-3}$, (b) Our results for $n_B = 0.4 fm^{-3}$ ($\Delta p_d = 6.78$), (c) Iwamoto results for $n_B = 1.4 fm^{-3}$, (d) Our results for $n_B = 1.4 fm^{-3}$ ($\Delta p_d = 10.29$).

Fig. 8.2: Three flavour d-decay for $\alpha_c = 0.1$ and s quark mass is 150 MeV; (a) Our results for $n_B = 1.4 fm^{-3}$, (b) Iwamoto results for $n_B = 1.4 fm^{-3}$ ($\Delta p_d = 0.067$), (c) Our results for $n_B = 0.4 fm^{-3}$, (d) Iwamoto results for $n_B = 0.4 fm^{-3}$ ($\Delta p_d = 0.39$).

Fig. 8.3: Three flavour s-decay for $\alpha_c = 0.1$ and s quark mass is 150 MeV; (a) Our results for $n_B = 1.4 fm^{-3}$, (b) Iwamoto results for $n_B = 1.4 fm^{-3}$ ($\Delta p_s = 1.613$), (c) Our results for $n_B = 0.4 fm^{-3}$, (d) Iwamoto results for $n_B = 0.4 fm^{-3}$ ($\Delta p_d = 9.719$).

TABLE 8.1

BARYON NUMBER DENSITY n_B, FERMI MOMENTA OF u-QUARK $p_f(u)$, d-QUARK $p_f(d)$, AND ELECTRON $p_f(e)$ FOR DIFFERENT α_c, WHERE $\Delta p_d = p_f(u) + p_f(e) - p_f(d)$.

α_c	n_B (fm^{-3})	$p_f(u)$ (MeV)	$p_f(d)$ (MeV)	$p_f(e)$ (MeV)	Δp_d (MeV)
	0.60	357.80	449.20	99.15	7.75
0.1	1.00	424.22	532.52	117.56	9.20
	1.40	474.57	595.79	131.51	10.29
	0.60	357.71	449.25	95.43	3.89
.05	1.00	424.11	532.65	113.15	4.61
	1.40	474.45	595.87	126.57	5.15

Fig. 8.2 shows the ϵ_d for three-flavour quark matter. It shows that ϵ_{dI} is 2 -3 orders of magnitude higher compared to our results. Here contrary to the two flavour case, the difference becomes more pronounced at higher densities. Fig. 8.3 shows the variation of ϵ_s with temperature. Here again, it is clear that ϵ_s is quite different from ϵ_{sI} but this difference is less compared to that between ϵ_d and ϵ_{dI}. For all the cases, the difference between our results and those using Iwamoto formula increases at higher temperatures. The Fermi momenta of quarks and electron for three- flavour case are given in Table 8.2. The study of all the figures and tables above reveals that the cases where Iwamoto formula agrees reasonably well with our results, Δp_d (or Δp_s) is much larger than the temperature. On the other hand, when this difference is smaller or comparable with the temperature, the Iwamoto formula overestimates the exact result by order of magnitude. In addition to these, Fig. 8.3 also shows that our results are about a factor of 2.5 lower than the Iwamoto results even at lower temperatures. We have found that this difference comes from the approximation involved in the calculation of matrix element.

TABLE 8.2

BARYON NUMBER DENSITY n_B, FERMI MOMENTA OF u-QUARK $p_f(u)$, d-QUARK $p_f(d)$, s-QUARK $p_f(s)$ AND ELECTRON $p_f(e)$ FOR DIFFERENT m_s AND DIFFERENT α_c, WHERE $\Delta p_d = p_f(u) + p_f(e) - p_f(d)$ AND $\Delta p_s = p_f(u) + p_f(e) - p_f(d)$

m_s (MeV)	α_c	n_B (fm^{-3})	$p_f(u)$ (MeV)	$p_f(d)$ (MeV)	$p_f(s)$ (MeV)	$p_f(e)$ (MeV)	Δp_d (MeV)	Δp_s (MeV)
150.0	0.1	0.60	356.99	360.06	353.86	3.33	0.26	6.46
		1.00	423.26	424.81	421.69	1.69	0.14	3.26
		1.40	473.49	474.27	472.72	0.84	0.06	1.61
	0.05	0.60	356.99	365.89	347.62	9.28	0.38	18.65
		1.00	423.26	430.29	415.98	7.33	0.30	14.61
		1.40	473.49	479.49	467.34	6.25	0.25	12.40
200.0	0.1	0.60	356.99	365.93	347.58	9.70	0.76	19.11
		1.00	423.26	429.10	417.25	6.34	0.50	12.35
		1.40	473.49	477.66	469.25	4.52	0.35	8.76
	0.05	0.60	356.99	374.26	337.85	18.00	0.73	37.14
		1.00	423.26	437.14	408.39	14.47	0.59	29.34
		1.40	473.49	485.45	460.90	12.46	0.50	25.05

Furthermore, Table 8.2. shows that for three flavour case electron chemical potential (which is same as $p_f(e)$ for massless electrons) becomes small ($< 1\ MeV$) for some values of α_c, n_B and m_s. In these cases, electrons are no longer degenerate. Clearly, for such cases the Iwamoto formula is not applicable. This point is missed in earlier calculations.

Our results are expected to have important implications for neutrino emissivity and quark star cooling rates because all the earlier calculations have used the Iwamoto formula and predicted large quark star cooling rates in comparison with the neutron star cooling rates for temperatures less than 1 MeV. Our results show that particularly, for three-flavour quark matter, the calculated emissivity is at least two orders of magnitude smaller than the one given by Iwamoto formula and therefore, the three-flavour quark star cooling rates are that much smaller. Hence, it is necessary to understand why Iwamoto formula fails.

To investigate the shortcoming of the Iwamoto formula, we consider the integral

$$I = \int \frac{d^3p_d d^3p_u d^3p_e d^3p_\nu}{\epsilon_d \epsilon_u \epsilon_e}$$
$$\times \delta^4(p_{d(s)} - p_u - p_e - p_\nu) n(\vec{p}_{d(s)})[1 - n(\vec{p}_u)][1 - n(\vec{p}_e)]. \quad (5.25)$$

Here, we have replaced the neutrino emission rate by unity and therefore I is essentially the phase space integral. Following the reasoning of Iwamoto, this integral should be proportional to T^5. Choosing the coordinate axes such that $\vec{p}_d$ is along z-axis and $\vec{p}_u$ is in $x - z$ plane and using the three-momentum $\delta-$function to perform electron and u-quark angle integrations, we get

$$I = 8\pi^2 \int \frac{p_d^2 dp_d p_u^2 dp_u p_e^2 dp_e d^3p_\nu}{\epsilon_d \epsilon_u \epsilon_e}$$

$$\times[\frac{\sqrt{1-x_u^2}}{p_e p_u(\sqrt{1-x_u^2}(p_d - p_\nu x_\nu) + p_\nu x_\nu \sqrt{1.-x_\nu^2}\cos\phi_\nu}]$$
$$\times\delta(\epsilon_d - \epsilon_u - \epsilon_e - \epsilon_\nu) n(\vec{p}_d)[1-n(\vec{p}_u)][1-n(\vec{p}_e)], \quad (5.26)$$

where $x_\nu = \cos\theta_\nu$ and $x_u = \cos\theta_u$ is determined by solving

$$p_u x_u = p_d - p_\nu x_\nu - [p_e^2 - p_u^2(1-x_u^2) - p_\nu^2(1-x_\nu^2)$$
$$-2p_u p_\nu \sqrt{(1-x_u^2)(1-x_\nu^2)}\cos\phi_\nu]^{1/2}. \quad (5.27)$$

The integral in Eq. (8.26) above is restricted to the momenta $|p_i - p_f(i)|$ few times T due to Fermi distribution functions and the energy $\delta-$function. Now, if we neglect the neutrino momentum in the $\delta-$functions, we get, $x_u = (p_d^2 + p_u^2 - p_e^2)/2p_d p_u$ and the factor in the square brackets of Eq. (8.26) becomes $1/p_d p_u p_e$.

Two points should be noted at this stage.

1. Generally, x_u is close to unity, so that $1-x_u^2$ is small. But, if Δp_d is of the order of T, $\sqrt{1-x_u^2}p_d$ can be comparable with T and p_ν and therefore p_ν cannot be neglected in the momentum $\delta-$functions. Particularly, the denominator in the square bracket of Eq. (8.26) cannot be approximated by $p_e p_u p_d \sqrt{1-x_u^2}$. Thus, if $p_d\sqrt{1-x_u^2} < p_\nu$, one would get a power of T from the denominator and I will not be proportional to T^5.

2. Secondly, the momenta may differ from the corresponding Fermi momenta by few times T in the integral. When $\Delta p_d \sim T$, there are regions in $p_d p_u p_e-$space where $x_u > 1$ and the rest of the integrand is not small. Clearly, these regions must be excluded from the integration as these values of x_u are unphysical. If one does not

put this restriction, as is done when one factorises angle and momentum integrals, the phase space integral will be overestimated (and wrong) when $\Delta p_d \sim T$.

The above discussion clearly shows why the integral in Eq. (8.26) should not be proportional to T^5 when $\Delta p_d \sim T$. In order to demonstrate this point, we have calculated the integral in Eq. (8.26) numerically and compared with the approximation where the neutrino momenta are neglected and the restriction imposed by x_u condition is not imposed. The calculation is done for $\alpha_c = 0.1$ and for two-flavour case. The results are shown in Fig. 8.4. In this figure, we also show the result for a case where the electron mass is taken to be 25 MeV. This is of course unphysical, but by adjusting the electron mass we can reduce Δp_d. The figure clearly shows that the approximate value of I is proportional to T^5 where as the exact integral is smaller than the approximate one at large T. Further more, for 25 MeV electron mass, the departure from T^5 sets in at smaller value of the temperature. This clearly shows that the departure is dependent on the value of Δp_d. Here we would like to mention that for some values of α_c and m_s , $p_f(e)$ is small and is of the order of T. This implies that electrons are no longer degenerate and deviation from the Iwamoto result is most pronounced.

In Eq. (8.26), we have dropped the matrix element of the weak interaction in the emissivity calculation (Eq. (8.21)). So, the discussion of preceeding paragraphs apply to the emissivity calculation as well. Therefore, it is now clear why the Iwamoto formula fails, when Δp_d (or Δp_s in case of weak interactions involving strange quarks) is close to the temperature of the quark matter.

Similar approximations have been used by other authors [164, 165]. In Ref. [164],

Fig. 8.4: Two flavour phase space integrals for $\alpha_c = 0.1$ (a) Without restriction on $\cos\theta_u$ for both electron mass m_e=0.0 and 25 MeV, (b) Exact integral for m_e=0.0, (c) Exact integral for m_e=25 MeV.

Fig. 8.5: $\frac{\epsilon_{d(s)I}}{\epsilon_{d(s)}}$ is plotted against x where $x = \frac{T}{\Delta p_{d(s)}}$. The fitted function is $f(x) = 1 + ax + bx^2 + cx^3$ where $a = -2.5$, $b = 100.$ and $c = 30$.

Burrows has calculated the neutrino emissivity for non-interacting quark matter from reverse beta decay. The exponent of T is 7 instead of 6 (Iwamoto), because of the partial restriction of the electron's phase space. Duncan *et al.* [165] computed the emissivity from reverse and direct beta decay for both interacting and non-interacting quark matter. They have reproduced the results of Burrows (T^7) [164] for non-interacting reverse beta decay and Iwamoto (T^6) [148] for interacting quark matter. In both these works, the effect of the finite neutrino momentum is included but rest of the calculation follows the approximation scheme of Iwamoto. Hence, their temperature dependence of emissivity formula is different from ours. Here, we would like to note that the departure from T^6 dependence of the emissivity essentially arises from the careful phase space integration. Since, similar approximation scheme is used to obtain the neutrino emissivity of neutron matter, it is possible that the emissivity calculated for neutron matter may also be overestimated when x is large.

Since the departure from the Iwamoto formula arises from the fact that $T/\Delta p_d$ (or $T/\Delta p_s$) is not small, it may be possible to fit the numerically calculated ϵ with a function of the form $\epsilon_I/f(x)$, where $x = T/\Delta p_d$ ($T/\Delta p_s$ for strange sector). The function $f(x)$ should be such that for small values of x it should approach unity. Choosing $f(x) = 1 + ax + bx^2 + cx^3$, we have fitted the calculated ϵ for a number of values of n_B, α_c and m_s for both s and d decay and obtained the values of a, b and c. The quality of fit is shown in Fig. 8.5 (since, for s- decay, as mentioned above, there is difference of a factor of 2.5 in Iwamoto and our results even at lower temperatures, the data points for s- decay, in Fig. 8.5., have been scaled accordingly). The values of a, b and c are -2.5, 100. and 30. respectively. It is clear from Fig. 8.5 that for $x \to 0$, Iwamoto results approache to

our values. Hence, our fitting is valid for any values of x.

5.3 Non-equilibrium neutrino emissivity of quark matter

The neutrino emission is a dominating mechanism of cooling of quark or neutron stars with the internal temperature exceeding $\simeq 10^8 K$. The quark stars cool much faster than the neutron stars. The neutrino emissivity formula [148, 162] of the quark stars are obtained under the assumption that the system is in equilibrium with respect to weak interactions. However, as long as matter is strongly degenerate, the number density of neutrinos is small and their presence in the matter can be neglected. Thus, the beta equilibrium (chemical equilibrium) stands as a good approximation, which implies the equality of anti-neutrino and neutrino emissivity. Of course, one can work even though the chemical equilibrium is not satisfied. Radial pulsations of quark stars have periods $\sim$ *milliseconds*, much shorter than the time scale of the beta reaction at $T < 10^{10}K \sim 1\ MeV$. Local compression as well as the rarefaction are very much important for non-equilibrium beta reactions and depend on the difference between the chemical potentials of u, $d(s)$ quarks and e electron, *i.e.*, $\delta\mu = \mu_{d(s)} - \mu_u - \mu_e$. The non-equilibrium beta reactions, induced by the radial pulsations of neutron and quark stars, were studied by a number of authors [130, 153]. Recently, Madsen [155] and Sawyer [151] calculated the bulk viscosity and damping rates by invoking the non-equilibrium condition in the quark star matter. However, later, Haensel [166] calculated the neutrino emissivity of non-equilibrium neutron star matter, where he found that the emissivity depends on the non-equilibrium conditions such as the difference in the chemical potential of neutron, proton and electron ($\delta\mu' = \mu_n - \mu_p - \mu_e$). Moreover, he showed that with increase of $\delta\mu'(-\delta\mu')$

the emissivity increases (decreases). So, it is interesting to see the non-equilibrium effects in the quark star matter.

In this section, we consider the rapid compression of the liquid interior in quark stars, which has two/three component quark matter. Such a situation is expected to occur during the gravitational collapse of the neutron stars or the quark stars in black hole [167], which would take place when the mass of an accreting neutron star exceeds the maximum allowable mass for the equilibrium configurations. Thus, a significant deviation from the chemical equilibrium is to be expected because of the shrinking of the stellar radius, which implies the monotonic increase of the average density in collapsing stars. Here we concentrate [168] on the characteristic features of the beta non-equilibrium neutrino spectra and compared with that of the beta equilibrium neutrino spectra.

Let us consider the neutrino emissivity of quark matter having two/three flavour degrees of freedom. Each of the constituents separately is a Fermi liquid in thermodynamic equilibrium.

For the degenerate two flavour quark matter, the simplest neutrino processes are the direct beta decay reactions

$$\begin{aligned} direct\ \beta_- &: d \rightarrow u + e^- + \bar{\nu}_e \\ direct\ \beta_+ &: u + e^- \rightarrow d + \nu_e. \end{aligned} \tag{5.28}$$

The distinction between β_- and β_+ processes will be discussed later on.

Moreover, the charge neutrality process of the two flavour quark matter is

$$2n_u - n_d - 3n_e = 0 \tag{5.29}$$

and the baryon density is defined as $n_B = (n_u + n_d)/3$. Since the matter is strongly degenerate, we write the Fermi momentum $p_{Fi} = (6\pi^2 n_i/g)^{1/3}$ by neglecting the thermal corrections, n_i being the number density of i-th particle and the degeneracy factor g is 6 for quark and 2 for electron. However, the non-equilibrium condition for the above reaction is

$$\mu_d - \mu_u - \mu_e = \delta\mu(\mu_{\nu_e} = \mu_{\bar{\nu}_e} = 0). \tag{5.30}$$

Here, we approximate the positive $\delta\mu$ for the direct β_- reaction (anti-neutrino emission) and that of the negative $\delta\mu$ for the direct β_+ reaction (neutrino emission).

Similarly, for the three flavour degenerate quark matter, the simplest neutrino process are the direct beta decay reaction, which occurs through d and s quarks. In addition to Eq. (8.28), one has also the following reactions :

$$\begin{aligned} &direct\ \beta_- : s \rightarrow u + e^- + \bar{\nu}_e \\ &direct\ \beta_+ : u + e^- \rightarrow s + \nu_e \end{aligned} \tag{5.31}$$

The equations for the charge neutrality and the baryon density are respectively

$$2n_u - n_d - n_s - 3n_e = 0 \tag{5.32}$$

and

$$n_B = (n_u + n_d + n_s)/3, \tag{5.33}$$

and due to the non-equilibrium condition, one can write

$$\mu_d - \mu_s = \delta\mu. \tag{5.34}$$

Moreover, here we again approximate the positive $\delta\mu$ for the direct β_- reaction (anti-neutrino emission) and the negative $\delta\mu$ for that of the direct β_+ reaction (neutrino emission).

Thus, in these approximations for both the two and the three flavour quark matter, the neutrino emissivity is almost same, *i.e.*, $\varepsilon_{\bar{\nu}_e}(-\delta\mu) \simeq \varepsilon_{\nu_e}(\delta\mu)$. In the chemical equilibrium, $\delta\mu = 0$, which implies $\varepsilon_{\bar{\nu}_e} \simeq \varepsilon_{\nu_e}$.

The energy momentum relation of s quark is approximated by Eq. (8.19) and that of u and d quarks by Eq. (8.20), *i.e.*, μ is replaced by E and p_F is replaced by p, in above Eqs. (8.19-8.20). This is reasonable since only the energies and the momenta close to the Fermi surface will contribute to the matrix element. The neutrino emissivity $\varepsilon_{d(s)}$ for $d(s)$ decay [148] is given by Eq. (8.21). The momentum delta function in Eq. (8.21) is used to integrate over the neutrino momentum p_ν. For the massless electrons, $E_e = p_e$ and the energy delta function is used to perform the integral over p_e. This gives

$$p_e = \frac{(E_{d(s)} - E_u)^2 - (p_{d(s)} - p_u)^2 - 2p_{d(s)}p_u(1 - cos\theta_u)}{(2p_u cos\theta_{ue} + 2p_{d(s)}cos\theta_e + 2E_{d(s)} - 2E_u)}, \tag{5.35}$$

where θ_u, θ_e and θ_{ue} are the angles between d and u, d and e and u and e respectively.

The chemical potentials and Fermi momenta thus obtained by employing the above non-equilibrium and charge neutrality conditions for two and three flavour matter are substituted in neutrino emissivity expression [Eq. 8.21] which subsequently yield

$$\begin{aligned} \varepsilon_{d(s)} = A_{d(s)} \int d^3p_{d(s)} d^3p_u p_e^2 d\Omega_e \frac{(p_{d(s)} \cdot p_\nu)(p_u \cdot p_e)}{E_u E_{d(s)} E_e} \\ \times n(\vec{p}_{d(s)})[1 - n(\vec{p}_u)][1 - n(\vec{p}_e)]. \end{aligned} \tag{5.36}$$

In the present case, the above emissivity has been evaluated numerically. The integral is five dimensional as all the angles are measured with respect to $d(s)$ quark. Throughout

the calculation, we have taken u, d, e, and ν masses to be zero, s quark mass m_s and strong coupling constant α_c to be 150 MeV and 0.1 respectively.

Recently [162], the neutrino emissivity of the degenerate quark matter was calculated exactly with the chemical equilibrium ($\delta\mu = 0.0$), which has been discussed in the previous section. It has been found that the neutrino emissivity results are in qualitative agreement with that of Iwamoto [148] for the two flavour quark matter, whereas for the three flavour quark matter Iwamoto's result overestimates the numerical values by nearly 2 orders of magnitude or more for d decay and agrees with that of the s-decay results within a factor of 3-4. Also, it was pointed out that the dependence of the temperature and the density on the neutrino emissivity is quite different from Iwamoto's result and is sensitive to the s quark mass.

Here, we use the same method [162] to calculate the neutrino emissivity by employing the non-equilibrium condition. For the two values of temperatures as well as baryon densities and different values of $\delta\mu$, the numerical values of the neutrino emissivity for two and three flavour quark matter are quoted in Table 8.3 and 8.4 respectively. The variation of the corresponding Fermi momenta with respect to the increase and decrease of $\delta\mu$ are presented in Table 8.5 and 8.6. We found here that the dependence of emissivity on n_B and T is rather different from that of the equilibrium condition ($\delta\mu = 0$).

For the two flavour quark matter, the anti-neutrino emissivity increases while that of the neutrino emissivity decreases on increasing $\delta\mu$ (same as increasing $-\delta\mu$ in the case of neutrino). For a constant temperature of $T = 0.4\ MeV$, the anti-neutrino emissivity depends on its exponent as $\varepsilon_{\bar{\nu}} \propto T^{3.02}$ to $T^{1.47}$ and on that of the baryon density as $\varepsilon_{\bar{\nu}} \propto n_B^{1.1}$ to $n_B^{1.22}$ for $\delta\mu$ varies from 1. to 2. MeV (0.005 to $0.01 fm^{-1}$) In a similar

TABLE 8.3

EMISSIVITY FOR TWO FLAVOUR QUARK MATTER. HERE $\epsilon_{\bar{\nu}}^{d}(\delta\mu)$ AND $\epsilon_{\bar{\nu}}^{d}(-\delta\mu)$ ARE THE EMISSIVITIES FOR d DECAY.

T	n_B	$\epsilon_{\bar{\nu}}^{d}(-\delta\mu)$	$\epsilon_{\bar{\nu}}^{d}(\delta\mu)$	$\delta\mu$
(MeV)	(fm^{-3})	$(erg/cm^3/s)$	$(erg/cm^3/s)$	fm^{-1}
0.4	0.8	$7.64\times10^{+27}$	$1.20\times10^{+31}$	0.005
	1.0	$9.59\times10^{+27}$	$1.54\times10^{+31}$	
	0.8	$7.22\times10^{+26}$	$4.50\times10^{+31}$	0.0075
	1.0	$9.02\times10^{+26}$	$5.80\times10^{+31}$	
	0.8	$3.75\times10^{+25}$	$1.36\times10^{+32}$	0.01
	1.0	$4.52\times10^{+25}$	$1.79\times10^{+32}$	
	0.8	$4.50\times10^{+29}$	$4.50\times10^{+29}$	0.00
	1.0	$5.70\times10^{+29}$	$5.70\times10^{+29}$	
0.5	0.8	$6.67\times10^{+28}$	$2.33\times10^{+31}$	0.005
	1.0	$8.41\times10^{+28}$	$3.02\times10^{+31}$	
	0.8	$1.12\times10^{+28}$	$7.10\times10^{+31}$	0.0075
	1.0	$1.42\times10^{+28}$	$9.29\times10^{+31}$	
	0.8	$1.58\times10^{+27}$	$1.86\times10^{+32}$	0.01
	1.0	$1.97\times10^{+27}$	$2.49\times10^{+32}$	
	0.8	$1.61\times10^{+30}$	$1.61\times10^{+30}$	0.00
	1.0	$2.05\times10^{+30}$	$2.05\times10^{+30}$	

TABLE 8.4

EMISSIVITY FOR THREE FLAVOUR QUARK MATTER. HERE $\epsilon_{\bar{\nu}}^{d}(\delta\mu)$ AND $\epsilon_{\bar{\nu}}^{d}(-\delta\mu)$ ARE THE EMISSIVITIES FOR d DECAY AND $\epsilon_{\bar{\nu}}^{s}(\delta\mu)$ AND $\epsilon_{\bar{\nu}}^{s}(-\delta\mu)$ ARE THE EMISSIVITIES FOR s DECAY.

T	n_B	$\epsilon_{\bar{\nu}}^{d}(\delta\mu)$	$\epsilon_{\bar{\nu}}^{d}(-\delta\mu)$	$\epsilon_{\bar{\nu}}^{s}(\delta\mu)$	$\epsilon_{\bar{\nu}}^{s}(-\delta\mu)$	$\delta\mu$
(MeV)	(fm^{-3})	$(erg/cm^3/s)$	$(erg/cm^3/s)$	$(erg/cm^3/s)$	$(erg/cm^3/s)$	MeV
	0.8	$1.54\times10^{+24}$	$4.46\times10^{+23}$	$6.47\times10^{+26}$	$8.52\times10^{+23}$	1.00
	1.0	$8.31\times10^{+23}$	$1.78\times10^{+23}$	$4.20\times10^{+26}$	$6.51\times10^{+23}$	
	0.8	$2.55\times10^{+24}$	$2.01\times10^{+23}$	$5.43\times10^{+27}$	$1.26\times10^{+22}$	2.00
	1.0	$1.51\times10^{+24}$	$6.51\times10^{+22}$	$3.00\times10^{+27}$	$9.87\times10^{+21}$	
0.2	0.8	$3.75\times10^{+24}$	$7.76\times10^{+22}$	$2.43\times10^{+28}$	$1.23\times10^{+20}$	3.00
	1.0	$2.53\times10^{+24}$	$1.82\times10^{+22}$	$1.07\times10^{+28}$	$9.75\times10^{+19}$	
	0.8	$8.72\times10^{+23}$	$8.72\times10^{+23}$	$3.43\times10^{+25}$	$3.43\times10^{+25}$	0.00
	1.0	$4.10\times10^{+23}$	$4.10\times10^{+23}$	$2.49\times10^{+25}$	$2.49\times10^{+25}$	
	0.8	$3.02\times10^{+25}$	$1.20\times10^{+25}$	$6.18\times10^{+27}$	$2.64\times10^{+26}$	1.00
	1.0	$1.94\times10^{+25}$	$6.93\times10^{+24}$	$3.20\times10^{+27}$	$1.58\times10^{+26}$	
	0.8	$4.50\times10^{+25}$	$7.08\times10^{+24}$	$2.07\times10^{+28}$	$4.08\times10^{+25}$	2.00
	1.0	$3.06\times10^{+25}$	$3.75\times10^{+24}$	$9.73\times10^{+27}$	$2.49\times10^{+25}$	
0.4	0.8	$6.49\times10^{+25}$	$3.93\times10^{+24}$	$5.49\times10^{+28}$	$5.45\times10^{+24}$	3.00
	1.0	$4.60\times10^{+25}$	$1.80\times10^{+24}$	$2.33\times10^{+28}$	$3.41\times10^{+24}$	
	0.8	$1.93\times10^{+25}$	$1.93\times10^{+25}$	$1.43\times10^{+27}$	$1.43\times10^{+27}$	0.00
	1.0	$1.19\times10^{+25}$	$1.19\times10^{+25}$	$8.15\times10^{+26}$	$8.15\times10^{+26}$	

way, the neutrino emissivity is proportional to the temperature as well as the baryon density exponents (for the constant temperature of $T = 0.4\ MeV$ as in the earlier case) as $\varepsilon_\nu \propto T^{9.72}$ to $T^{16.92}$ and $n_B^{1.03}$ to $n_B^{0.84}$ for $\delta\mu$ = -1. to -2.MeV (0.005 to 0.01 fm^-1). But for the equilibrium case ($\delta\mu = 0$), $\varepsilon_{\bar{\nu}\nu} \propto T^{5.3-5.9}$ and $\propto n_B^{1.03}$ as has been observed earlier [162].

In addition, for the three flavour quark matter, the behaviour of the anti-neutrino as well as the neutrino emissivity is qualitatively same due to the variation in $\delta\mu$(-$\delta\mu$ in the case of neutrino), as is seen above in the case of two flavour quark matter. But unlike the two flavour quark matter, the anti-neutrino and the neutrino emissivity is inversely proportional to the baryon density. The variation in the exponents here (for $T = 0.4\ MeV$ in both cases) is, $\varepsilon_{\bar{\nu}} \propto T^{4.55}$ to $T^{4.19}$ and $n_B^{-1.96}$ to $n_B^{-2.16}$ for the d decay, and $\propto T^{2.93}$ to $T^{1.12}$; and $n_B^{-2.95}$ to $n_B^{-3.85}$ for the s decay when one changes $\delta\mu$ from 1. to 2. MeV. Similarly, $\varepsilon_\nu \propto T^{5.29}$ to $T^{6.88}$; and $n_B^{-2.48}$ to $n_B^{-3.51}$ for the d decay, and $\propto T^{7.92}$ to $T^{15.09}$; and $n_B^{-2.31}$ to $n_B^{-2.11}$ for the s decay due to variation in $\delta\mu$= –1. to –2. MeV. But the results in the equilibrium case ($\delta\mu = 0$) which has already been shown in the literature [162] is $\varepsilon_{\bar{\nu}\nu} \propto T^{4.86-4.47}$; $\propto n_B^{-2.16}$ for the case of d decay and $\propto T^{5.03-5.38}$, $n_B^{-2.53}$ for s decay with $m_s = 150\ MeV$ and $\alpha_c = 0.1$.

Therefore, it is obvious from the above numerical results that the exponent of the temperature decreases but that of the baryon density increases in the anti-neutrino emissivity case compared to the equilibrium condition. But for the case of neutrino emissivity, the exponent of the temperature increases and the baryon density exponent decreases. Thus it is clear that the changes on the neutrino emissivity as well as the anti-neutrino emissivity is due to the non-equilibrium condition where $\delta\mu \neq 0$. The change in the emissivity

($\varepsilon_{\bar{\nu}}(\varepsilon_{\nu})$) due to the variation in $\delta\mu(-\delta\mu)$ for $T = 0.4$ MeV is shown in Table 8.3 and 8.4 for two flavour and three flavour case, which has also been extended to $T = 0.5$ MeV (Table 8.3 and 8.4). Moreover, it is seen from Table 8.5 and 8.6 that the difference in the chemical potential affects the density distribution of particles for both two and three flavour quark matter case. So, when $\delta\mu(-\delta\mu)$ increases from 0.0 to 0.01 fm^{-1}, the Fermi momentum of u quark and e increase (decrease), and that of d quark decreases (increases) with respect to the chemical equilibrium Fermi momenta of u, d and e for two flavour quark matter. But for the three flavour quark matter case (for d and s decay), the chemical equilibrium Fermi momentum of u quark remains the same, and that of d quark and e decrease (increase) and s increases (decreases) as one changes $\delta\mu(-\delta\mu)$ from 0.0 to 2.0 MeV. Hence, in comparison to the equilibrium case, the neutrino emissivity decreases monotonically, whereas that of the anti-neutrino emissivity increases monotonically for both the two and the three flavour quark matter.

TABLE 8.5

BARYON NUMBER DENSITY n_B, FERMI MOMENTA OF u-QUARK $p_F(u)$, d-QUARK $p_F(d)$ AND ELECTRON $p_F(e)$.

$\delta\mu$ fm^{-1}	n_B (fm^{-3})	$p_F(u)$ (MeV)	$p_F(d)$ (MeV)	$p_F(e)$ (MeV)
	0.8	393.81	494.40	109.13
0.0	1.0	424.22	532.58	117.56
	0.8	393.89	494.35	112.16
0.0075	1.0	424.30	532.53	120.59
	0.8	393.92	494.34	113.17
0.01	1.0	424.33	532.51	121.60
	0.8	393.74	494.45	106.09
-.0075	1.0	424.14	532.63	114.52
	0.8	393.71	494.47	105.08
-.01	1.0	424.12	532.64	113.50

TABLE 8.6

BARYON NUMBER DENSITY n_B, FERMI MOMENTA OF u-QUARK $p_F(u)$, d-QUARK $p_F(d)$, s-QUARK $p_F(s)$ AND ELECTRON $p_F(e)$ FOR $m_s = 150\ MeV$ AND $\alpha_c = 0.1$.

$\delta\mu$ (MeV)	n_B (fm^{-3})	$p_F(u)$ (MeV)	$p_F(d)$ (MeV)	$p_F(s)$ (MeV)	$p_F(e)$ (MeV)
	0.8	392.92	395.08	390.73	2.35
0.0	0.1	423.26	424.81	421.69	1.69
	0.8	392.92	394.62	391.20	1.85
1.0	0.1	423.26	424.35	422.16	1.18
	0.8	392.92	394.15	391.68	1.34
2.0	0.1	423.26	423.88	422.63	0.68
	0.8	392.92	395.55	390.25	2.86
-1.0	0.1	423.26	425.28	421.21	2.19
	0.8	392.92	396.01	389.77	3.36
-2.0	0.1	423.26	425.74	420.74	2.70

Chapter 6

Summary and Conclusion

In this thesis, we have investigated the behaviour of matter at very high density using a relativistic Lagrangian description. The Lagrangian chosen by us corresponds to the chiral sigma model. This approach is considered to be a "good" low energy limit of quantum chromo dynamics. Although there have been a few previous calculations along this line, a detailed and consistent field theoretical approach has been lacking. The calculations presented in this thesis are aimed at such a detailed study. We have extended these calculations for the case of finite temperatures ($\leq 15\ MeV$). The results are expected to find application in stellar collapse calculations. In addition, we have dealt with the following subjects : (1) Phase transition to quark matter and the possible formation of strangelets at high densities and (2) astrophysical applications of our results, to (a) structure and radial oscillation of nonrotating neutron stars and (b) the neutrino emissivity of quark matter with an improved calculation of phase space integrals involved.

The highlights and main results of this thesis can be summarized as follows :

1. The energy per nucleon of cold nuclear matter ($k_B T = 0$), derived by us using chiral sigma model, is in good agreement with the preliminary estimates inferred

from heavy-ion collision data [57] in the density range between one to four times the nuclear saturation density (n_s).

2. For a system of high density nuclear matter, based on the chiral sigma model, we find that a strict first order phase transition to $(u,\ d,\ s)$ quark matter is not favoured. This does not, of course, preclude a phase transition of second order. However, we have not investigated the latter problem.

3. The mass formulae for finite lumps of strange quark matter with $u,\ d$ and s quarks and non-strange quark matter (u and d) are derived in a non-relativistic approach, taking into account the finite size effects such as surface and curvature. We find that there is a good possibility for the formation of metastable strangelets of large mass detectable in experiment. This is important since the detection of strangelets may be the most unambiguous way to confirm the formation of quark-gluon plasma in heavy ion collision experiment.

4. The maximum mass for stable neutron stars predicted by our equation of state for $(n,\ p,\ e)$ matter is 2.59 times the solar mass. The corresponding radius (R), crustal length (Δ) and surface red shift ratio (α) are 14.03 km, 1.0 km and 0.674 respectively. The maximum moment of inertia is $4.79 \times 10^{45}\ g\ cm^2$. These suggest that our equation of state for neutron star matter is comparatively "stiff". This is reflected in the value of the maximum mass of neutron stars, which is the largest for the present model as compared to other available field theoretical equation of state models. It may be mentioned here that observational evidence in favour of a stiff equation of state comes from the identification by Trümper *et al*. [64] of the 35 day

cycle of the pulsating X-ray source Her X-1 as originating in free precession of the rotating neutron star (Pines [65] for a discussion). For a 1.4 times the solar mass neutron star configuration, we get : $R = 14.77\ km$, $I = 2.15 \times 10^{45}\ g\ cm^2$, Δ=3.0 km and the red shift ratio (at the surface) $\alpha = 0.85$. The corresponding central density is $4.06 \times 10^{14}\ g\ cm^{-3}$.

5. The neutrino emissivity from two and three flavour quark matter is numerically calculated and compared with the result given by Iwamoto [148]. We find that the emissivity is smaller than Iwamoto's result by about two orders of magnitude when $p_f(u)+p_f(e)-p_f(d(s))$ is comparable to the temperature. We attribute this to the severe restriction imposed by momentum conservation on the phase space integral. An alternative formula for the neutrino emissivity, which is valid when the quarks and electrons are degenerate and any values of $p_f(u)+p_f(e)-p_f(d(s))$ is obtained by us.

Bibliography

[1] W. Baade and F. Zwicky, *Phys. Rev.* **45** (1934) 138.

[2] J. R. Oppenheimer and G. M. Volkoff, *Phys. Rev.* **55** (1939) 374.

[3] A. Hewish, S. J. Bell, J. D. H. Pilkington, P. F. Scott and R. A. Collins, *Nature* **217** (1968) 709.

[4] C. Gale, G. M. Welke, M. Prakash, S. J. Lee and S. Das Gupta, *Phys. Rev. C* **41** (1990) 1545.

[5] V. Canuto, *Ann. Rev. Astr. Astrophys.* **12** (1974) 167.

[6] G. Baym and C. Pethick, *Ann. Rev. Astr. Astrophys.* **17** (1979) 415.

[7] B. Datta and M. A. Alpar, *Astron. Atrophys.* **275** (1993) 210.

[8] J. Zimanyi and S. A. Moszkowski, *Phys. Rev. C* **42** (1990) 1416.

[9] B. D. Day and R. B. Wiringa, *Phys. Rev. C* **32** (1985) 1057.

[10] V. R. Pandharipande, *Nucl. Phys. A* **174** (1971) 641.

[11] R. B. Wiringa, V. Fiks and A. Fabrocini, *Phys. Rev. C* **38** (1988) 1010.

[12] R. V. Reid, *Ann. Phys.* **50** (1968) 411 (R).

[13] G. Baym and C. Pethick, *Ann. Rev. Nucl. Sci.* **25** (1975) 27.

[14] V. Canuto, *Ann. Rev. Astr. Astrophys.* **13** (1975) 335.

[15] V. R. Pandharipande and H. A. Bethe, *Phys. Rev. C* **7** (1973) 1312.

[16] V. R. Pandharipande and R. B. Wiringa, *Nucl. Phys. A* **266** (1976) 269.

[17] H. A. Bethe and M. B. Johnson, *Nucl. Phys. A* **230** (1974) 1.

[18] R. C. Malone, M. B. Johnson and H. A. Bethe, *Astrophys. J.* **199** (1975) 741.

[19] V. R. Pandharipande and R. A. Smith, *Nucl. Phys. A* **237** (1975) 507.

[20] A. M. Green and P. Haapakoski, *Nucl. Phys. A* **221** (1974) 429.

[21] R. A. Smith and V. R. Pandharipande, *Nucl. Phys. A* **256** (1975) 327.

[22] A. M. Green and J. A. Niskanen, *preprint, University of Helsinki, Finland* (1975).

[23] V. R. Pandharipande and R. A. Smith, *Phys. Lett. B* **59** (1975) 15.

[24] J. D. Walecka, *Ann. Phys.* **83** (1974) 491.

[25] V. R. Pandharipande and H. A. Bethe, *Phys. Rev. C* **7** (1973) 1312.

[26] B. Friedman and V. R. Pandharipande, *Nucl. Phys. A* **361** (1981) 502.

[27] V. R. Pandharipande and R. B. Wiringa, *Rev. Mod. Phys.* **51** (1979) 821.

[28] J. M. Lattimer and D. G. Ravenhall, *Astrophys. J.* **223** (1978) 314.

[29] I. E. Lagaris and V. R. Pandharipande, *Nucl. Phys. A* **359** (1981) 349.

[30] I. E. Lagaris and V. R. Pandharipande, *Nucl. Phys. A* **334** (1980) 217.

[31] N. K. Glendenning, *Ann. Phys.* **149** (1983) 22.

[32] B. D. Serot and J. D. Walecka, *The relativistic nuclear many-body problem, in Advanced in Nucl. Phys.* **16** (1986).

[33] N. K. Glendenning, *Astrophys. J.* **293** (1985) 470.

[34] S. A. Moszkowski, *Phys. Rev. D* **9** (1974) 1613.

[35] N. K. Glendenning, *Nucl. Phys. A* **493** (1989) 521.

[36] J. I. Kapusta and K. A. Olive, *Phys. Rev. Lett.* **64** (1990) 13.

[37] J. D. Alonso and J. M. I. Cabanell, *Astrophys. J.* **291** (1985) 308.

[38] G. Baym, H. A. Bethe and C. J. Pethick, *Nucl. Phys. A* **175** (1971) 225.

[39] H. Pagels, *Phys. Rev.* **16** (1975) 219.

[40] N. K. Glendenning, *Ann. Phys.* **168** (1986) 246.

[41] N. K. Glendenning, *Nucl. Phys. A* **480** (1988) 597.

[42] M. Prakash and T. L. Ainsworth, *Phys. Rev. C* **36** (1987) 346.

[43] E. Baron, J. Cooperstein and S. Kahana, *Phys. Rev. Lett.* **55** (1985) 126; *Nucl. Phys. A* **440** (1985) 744.

[44] J. Cooperstein, *Phys. Rev. C* **37** (1988) 786.

[45] A. Rosenhauer *et al.*, *Nucl. Phys. A* **540** (1992) 630.

[46] A. J. Sierk and J. R. Nix, *Phys. Rev. C* **22** (1980) 1920.

[47] H. Stocker and W. Greiner, *Phys. Reports* **137** (1986) 277.

[48] N. K. Glendenning, F. Weber and S. A. Moszkowski, *Phys. Rev. C* **45** (1992) 844.

[49] A. D. Jackson, M. Rho and E. Krotscheck, *Nucl. Phys. A* **407** (1985) 495.

[50] T. L. Ainsworth, E. Baron, G. E. Brown, J. Cooperstein and M. Prakash, *Nucl. Phys. A* **464** (1987) 740.

[51] M. Gell-Mann and M. Levy, *Nuovo Cimento* **16** (1960) 705.

[52] T. D. Lee and G. C. Wick, *Phys. Rev. D* **9** (1974) 2291.

[53] J. Boguta, *Phys. Lett. B* **128** (1983) 19.

[54] T. Matsui and B. D. Serot, *Ann. Phys.* **144** (1982) 107.

[55] P. K. Sahu, R. Basu and B. Datta, *Astrophys. J.* **416** (1993) 267.

[56] C. Horowitz and B. Serot, *Nucl. Phys. A* **464** (1987) 613.

[57] R. Stock, *Nature* **337** (1989) 319.

[58] G. Baym, in *Neutron Stars : Theory and Observation*, edited by J. Ventura and D. Pines (Kluwer Academic) (1991) 26.

[59] J. Ellis, J. I. Kapusta and K. A. Olive, *Nucl. Phys. B* **348** (1991) 345.

[60] P. Moller, W. D. Myers, W. J. Swiatecki and J. Treiner, *Atomic Data Nucl. Data Tables* **39** (1988) 225.

[61] N. K. Glendenning, B. Banerjee and M. Gyulassy, *Ann. Phys.* **149** (1983) 1.

[62] B. D. Serot, *Phys. Lett. B* **86** (1979) 146.

[63] S. A. Chin, *Ann. Phys.* **108** (1977) 301.

[64] J. Trumper, P. Kahabka, H. Ogelman, W. Pietsch and W. Voges, *Astrophys. J.* **300** (1986) L63.

[65] D. Pines, *in High Energy Phenomena around Collapsed Stars*, edited by F. Pacini (Dordrecht : Reidel), (1987) 193.

[66] J. Cooperstein and J. Wambach, *Nucl. Phys. A* **420** (1984) 591; A. Ray, S. M. Chitre and K. Kar, *Astrophys. J.* **285** (1984) 766.

[67] K. A. Van Riper and S. A. Bludman, *Astrophys. J.* **213** (1977) 239.

[68] W. D. Arnett, *Astrophys. J.* **218** (1977) 815; D. K. Nadyozhin, *Astrophys. Space. Sci.* **49** (1977) 399; **51** (1977) 283.

[69] J. M. Lattimer and F. D. Swesty, *Nucl. Phys. A* **535** (1991) 331.

[70] H. A. Bethe, G. E. Brown, J. Applegate and J. M. Lattimer, *Nucl. Phys. A* **334** (1979) 487.

[71] J. D. Walecka, *Phys. Lett. B* **59** (1975) 109.

[72] R. L. Bowers, A. M. Gleeson and J. W. Wheeler, *Astrophys. J.* **213** (1977) 351; R. L. Bowers, A. M. Gleeson, R. D. Pedigo and J. W. Wheeler, *Phys. Rev. D* **15** (1977) 2125; R. A. Freedman, *Phys. Lett. B* **71** (1977) 369.

[73] M. Barranco and J. Treiner, *Nucl. Phys. A* **351** (1981) 269.

[74] J. K. Zhang and D. S. Onley, *Phys. Rev. C* **44** (1991) 2230.

[75] B. Datta and P. K. Sahu, *Phys. Lett. B* **318** (1993) 277.

[76] E. Fermi, *Progr. Theor. Phys.* **5** (1950) 570.

[77] I. Ya. Pomeranchuk, *Doklady Akad. Nauk*, **78** (1951) 889. (in Russian).

[78] L. D. Landau and Izvestiya *AN SSSR. Ser. Fiz.* **17** (1953) 51 (in Russian); *Collected papers of Landau, Gordon and Breach*, No. **4**, (1965).

[79] A. D. Sakharov, *Soviet. Phys. -JETP Lett.* **5** (1967) 32.

[80] A. M. Polyakov, *Phys. Lett. B* **82** (1979) 247.

[81] L. Susskind, *Phys. Rev. D* **20** (1979) 2610.

[82] J. Engels *et al.*, *Nucl. Phys. B* **205** (1982) 545.

[83] C. P. Singh, *Phys. Rep.* **236** (1993) 147.

[84] D. K. Srivastava, J. Alam, S. Chakrabarty, S. Raha and B. Sinha, *Ann. Phys.* **228** (1993) 104; D. K. Srivastava and C. Gale, *Phys. Lett. B* **319** (1993) 407.

[85] A. Chodos, R. L. Jaffe, C. B. Thorn and V. Weisskopf, *Phys. Rev. D* **9** (1974) 3471.

[86] L. D. Landau and E. M. Lifshitz, *Statistical Physics, Pergamon Press*, (1968).

[87] K. Wilson, *Phys. Rev. D* **10** (1974) 2445.

[88] E. V. Shuryak, *Phys. Rep.* **115** (1984) 153.

[89] J. Engels, F. Karsch, I. Montvay and H. Satz, *Phys. Lett. B* **101** (1981) 89; **102** (1981) 332.

[90] J. Engels, F. Karsch and H. Satz, *Nucl. Phys. B* **205** (1982) 239.

[91] F. Karsch, *Nucl. Phys. B* **205** (1982) 285.

[92] T. Celik, *et al.*, *Phys. Lett. B* **125** (1983) 411.

[93] T. Celik, *et al.*, *Phys. Lett. B* **129** (1983) 323.

[94] T. Banks and A. Ukawa, *Nucl. Phys. B* **225** (1983) 145.

[95] G. Baym and S. A. Chin, *Phys. Lett. B* **62** (1976) 241.

[96] G. Chapline and M. Nauenberg, *Phys. Rev. D* **16** (1977) 450.

[97] E. Farhi and R. L. Jaffe, *Phys. Rev. D* **30** (1984) 2379.

[98] P. Haensel, J. L. Zdunick and R. Schaeffer, *Astron. Astrophys.* **160** (1986) 121.

[99] E. Witten, *Phys. Rev. D* **30** (1984) 272.

[100] H. A. Bethe, G. E. Brown and J. Cooperstein, *Nucl. Phys. A* **462** (1987) 791.

[101] F. R. Brown *et al.*, *Phys. Rev. Lett.* **65** (1990) 2491.

[102] G. E. Brown, H. A. Bethe and P. M. Pizzochero, *Phys. Lett. B* **263** (1991) 337.

[103] A. Goyal and J. D. Anand, *Phys. Rev. D* **42** (1990) 992.

[104] A. R. Bodmer, *Phys. Rev. D* **4** (1971) 1601.

[105] S. A. Chin and A. K. Kerman, *Phys. Rev. Lett.* **43** (1979) 1292.

[106] B. Freedman and L. McLerran, *Phys. Rev. D* **17** (1978) 1109.

[107] M. S. Berger and R. L. Jaffe, *Phys. Rev. C* **35** (1987) 213; **44** (1991) 566(E).

[108] I. Mardor and B. Svetitsky, *Phys. Rev. D* **44** (1991) 878.

[109] J. Madsen, *Phys. Rev. Lett.* **70** (1993) 391; *Phys. Rev. D* **47** (1993) 5156.

[110] A. De Rujula, H. Georgi and S. L. Glashow, *Phys. Rev. D* **12** (1975) 147; N. Isgur and G. Karl, *Phys. lett. B* **72** (1977) 109; **74** (1978) 353; *Phys. Rev. D* **18** (1978) 4187; **20** (1979) 1191; F. E. Close, *An Introduction to Quarks and Partons* (Academic, New York, 1979); *Baryon 1980*, Proceedings of the International Conference on Baryon Resonances, Toronto, Canada, edited by N. Isgur (University of Toronto,Toronto, 1980).

[111] L. Satpathy, P. K. Sahu and V. S. Uma Maheswari, *Phys. Rev. D* **49** (1994) 4642.

[112] R. Balian and C. Bloch, *Ann. Phys. (N.Y.)* **60** (1970) 401.

[113] A. Chodos, R. L. Jaffe, K. Johnson and C. B. Thorn, *ibid* **10** (1974) 2599.

[114] D. Vautherin and D. M. Brink, *Phys. Rev. C* **5** (1972) 626.

[115] R. K. Bhaduri, *Models of the Nucleon*, (Addision- Wesley, Reading, MA, 1988), p.45.

[116] R. L. Jaffe, in *Strange Quark Matter in Physics and Astrophysics*, Proceeding of the International Workshop, Aarhus, Denmark, 1991, edited by J. Madsen and P. Haensel [*Nucl. Phys. B (Proc. Suppl.)* **24** (1991) 8].

[117] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation (San Francisco. Freeman)* (1970).

[118] J. B. Hartle, *Astrophys. J.* **150** (1967) 1005.

[119] J. W. Negele and D. Vautherin, *Nucl. Phys. A* **207** (1973) 298.

[120] G. Baym and C. J. Pethick and P. G. Sutherland, *Astrophys. J.* **170** (1971) 299.

[121] R. P. Feynman, N. Metropolis and E. Teller, *Phys. Rev.* **75** (1949) 1561.

[122] F. Nagase, *PASP* **41** (1989) 1.

[123] A. G. W. Cameron, *Nature* **205** (1965) 787.

[124] D. W. Meltzer and K. S. Thorne, *Astrophys. J.* **145** (1966) 514.

[125] K. S. Thorne, *Astrophys. J.* **158** (1969) 1; **158** (1969) 997.

[126] B. K. Harrison, K. S. Thorne and M. Wakono and J. R. Wheeler, in Gravitation Theory and Gravitational Collapse, *The University of Chicago Press* (1965) p.108; S. L. Shapiro and S. A. Teukolsky, in Black holes, White dwarfs, and Neutron stars, *John Wiley and Sons, Inc.* (1983) p.42.

[127] J. A. Wheeler, *Ann. Rev. Astron. Astrophys.* **4** (1966) 393.

[128] W. Y. Chau, *Astrophys. J.* **147** (1967) 664.

[129] F. Occhionero, *Mem. Soc. Astron. Ital.* **39** (1968) 351.

[130] C. Cutler, L. Lindblom and R. J. Splinter, *Astrophys. J.* **363** (1990) 603.

[131] C. Cutler and L. Lindblom, *Astrophys. J.* **385** (1992) 630.

[132] Y. Kojima, *Astrophys. J.* **414** (1993) 247.

[133] S. Chandrasekhar, *Astrophys. J.* **140** (1964) 417.

[134] B. Datta, P. K. Sahu, J. D. Anand and A. Goyal, *Phys. Lett. B* **283** (1992) 313.

[135] D. Ivanenko and D. F. Kurdgelaidze, *Lett. Nuovo Cimento* **2** (1969) 13.

[136] N. Itoh, *Prog. Theor. Phys.* **44** (1970) 291.

[137] B. D. Keister and L.S. Kisslinger, *Phys. Lett B* **64** (1976) 117.

[138] B. A. Freedman and L. D. McLerran, *Phys. Rev. D* **16** (1977) 1169.

[139] C. Alcock, E. Farhi and A. Olinto, *Astrophys. J.* **310** (1986) 261.

[140] K. Brecher, *Astrophys. J.* **215** (1977) L17; S. K. Ghosh and P. K. Sahu., *Int. Jour. Mod. Phys. E* **2** (1993) 575.

[141] L. Lindblom, *Astrophys. J.* **278** (1984) 364.

[142] L. Lindblom, *Astrophys. J.* **303** (1986) 146.

[143] J. L. Friedman, J. R. Ipser and L. Parker, *Astrophys. J.* **304** (1986) 115.

[144] N. K. Glendenning, *J. Phys. G : Nucl. Part. Phys.* **15** (1989) L 255.

[145] N. K. Glendenning, *Phys. Rev. Lett.* **63** (1989) 2629.

[146] M. Prakash, E. Baron and M. Prakash, *Phys. Lett. B* **243** (1990) 175.

[147] J. L. Zdunick and P. Haensel, *Phys. Rev. D* **42** (1990) 710.

[148] N. Iwamoto, *Phys. Rev. Lett.* **44** (1980) 1637; *Ann. Phys.* **141** (1982) 1.

[149] R. C. Duncan, S. L. Shapiro and I. Wasserman, *Astrophys. J.* **267** (1983) 358.

[150] B. Datta, S. Raha and B. Sinha, *Mod. Phys. Lett. A* **3** (1988) 1385.

[151] R. F. Sawyer, *Phys. Lett. B* **233** (1989) 412.

[152] P. Haensel and J. L. Jerzak, *Acta Phys. Pol. B* **20** (1989) 141.

[153] P. Haensel, J. L. Zdunick and R. Schaeffer, *Astron. Astrophys.* **217** (1989) 137.

[154] V. Baluni, *Phys. Lett. B* **72** (1978) 381; *Phys. Rev. D* **17** (1978) 2092.

[155] J. Madsen, *Phys. Rev. D* **46** (1992) 3290.

[156] J. N. Bahcall and R. A. Wolf, *Phys. Rev. B* **140** (1965) 1452.

[157] S. Tsuruta and A. G. W. Cameron, *Nature* **207** (1965) 364.

[158] K. Nomoto and S. Tsuruta, *Astrophys. J.* **312** (1987) 711.

[159] S. Tsuruta, *Comments Ap.* **11** (1986) 151 (and references therein).

[160] H. -Y. Chiu and E. E. Salpeter, *Phys. Rev. Lett.* **12** (1964) 413.

[161] J. M. Lattimer, C. J. Pethick, M. Prakash and P. Haensel, *Phys. Rev. Lett.* **66** (1991) 2701.

[162] S. K. Ghosh, S. C. Phatak and P. K. Sahu, *Mod. Phys. Lett. A* (1994) (in press).

[163] G. Baym and S. A. Chin, *Nucl. Phys. A* **262** (1976) 527.

[164] A. Burrows, *Phys. Rev. Lett.* **44** (1980) 1640.

[165] R. C. Duncan, S. L. Shapiro and I. Wasserman, *Astrophys. J.* **278** (1984) 806.

[166] P. Haensel, *Astron. Astrophys.* **262** (1992) 131.

[167] E. Gourgoulhon, P. Haensel, S. Bonazzola and J. A. Marck 1992, in : *Proc. Tenth Seminar in Relativistic Astrophysics and Gravitation, Potsdam, October 20-26, 1991 World Scientific* (to be published).

[168] P. K. Sahu, *Int. Jour. Mod. Phys. E* **2** (1993) 647.